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THE HIGH FREQUENCY SPECTRUM OF TUNGSTEN

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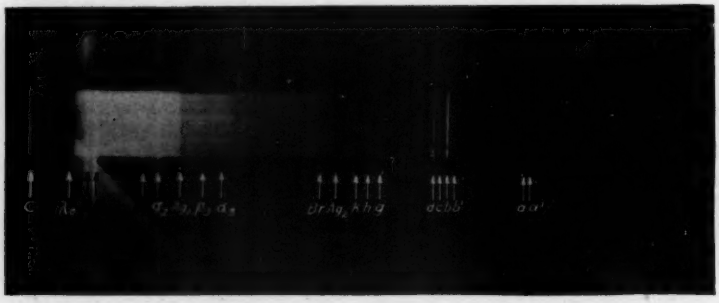
Mosely[1] has shown that the high frequency spectrum given out by the target of an X-ray tube consists of two series of lines superimposed upon a continuous spectrum. The lines, which are known as the K and L series, respectively, are characteristic of the material of the target. Mosely measured the wave lengths of most of the K lines for elements having atomic weights between aluminum and silver, and of the L lines for elements from calcium to gold, and showed that for all the lines measured the square roots of the frequencies of corresponding lines are proportional to the atomic numbers of the elements emitting them. Malmer[2] has added to this list the K lines of six more elements between silver and lanthanum, and W. H. Bragg[3] and others have studied in great detail the lines of a few of these elements, especially rhodium and platinum.

The continuous or band spectrum was observed qualitatively by Mosely (l.c.), and its short wave-length limit at different voltages measured by Duane and Hunt,[4] who found this limiting frequency, ν_{max}, to be exactly proportional to the voltage on the tube. and given accurately by the quantum relation $h\nu_{max} = eV$, where V is the voltage on the tube, e the charge of an electron, and h Planck's constant.

The spectrum of tungsten is of special interest on account of its use as target material in X-ray tubes, and it has been the subject of several recent investigations. Barnes[5] measured the L lines, but was unable to find any K lines, although his voltage, 96,000, was sufficiently high for their excitation. Gorton[6] also measured the L lines. Rutherford, Barnes and Richardson,[6] using the coefficient of absorption method,

measured the effective wave-length of the 'end radiation,' i.e., the short wave-length limit of the continuous spectrum, for different voltages up to 180,000, and found that this minimum wave-length did not decrease continuously with increase of voltage, but approached asymptotically a limiting value of 0.172 angstrom units. As will be shown below, the wave-lengths found by the spectrometer are much shorter, and do not appear to approach any limiting value.

The spectrum shown in figure 1 was taken in the usual manner with a rock salt crystal in continuous slow rotation, photographic plate stationary at 19.13 cm. distance from the crystal, collimating slits 0.2 mm. wide and 20 cm. apart, with a Coolidge tube running at 1 milli-



WAVE-LENGTHS OF LINES SHOWN IN FIG. 1

L = Designation of line, D = Distance from C in cm., λ = Wave-length in angstroms

L	λ_0	β_1	α_1	β_2	α_2		Ag$_1$	β_3	α_3		Br
D	0.964	1.332	1.496	2.614	2.932	2.994	3.332	3 948	4.386	4.494	6.524
λ	0.142	0.196	0.219	0.192	0.215	0.220	0.488	0.192	0.212	0 217	0.463

L	Ag$_2$	k	h		g	d	c	b	b'	a	a'
D	6.886	7.368	7.604	7.662	7.884	9 074	9.224	9.382	9.558	11.10	11.19
λ	0 487	1.033	1.065	1.073	1.100	1.242	1.260	1.280	1.300	1.468	1.480

ampere and 100,000 volts constant potential. The horizontal band gives the reflection from the cubic (100) planes of the crystal, those at 45° from the dodecahedral (110) planes, and those between from the tetrahexahedral (210) planes, etc.

The wave-lengths of the lines and bands in the horizontal strip are given in the accompanying table. C, in the photograph, is the undeviated central beam, diminished in intensity by passage through a lead strip 2 mm. thick and 5 mm. wide, placed in front of the plate. The shadow of this strip extends only one fourth of the distance from C to λ_0 and is not visible on the plate.

The short wave-length limit of the spectrum, marked λ_0, is the shortest wave-length that is produced by electrons of velocity corresponding to the operating voltage (100,000) and its value, 0.142 angstrom units, agrees very closely with the value given by Duane and Hunt's formula $eV = h\nu_{max} = hc/\lambda_{min}$. I have already shown[7] that the proportionality between frequency and voltage holds accurately up to 100,000 volts, and these measurements have since been extended, with less accuracy, up to 150,000 volts. The shortest wave-length so far observed is 8×10^{-10} cm., or 0.08 A.U.

The lines marked $\alpha_1\beta_1$, $\alpha_2\beta_2$, and $\alpha_3\beta_3$ are the first, second, and third order reflections respectively of the K_α and K_β lines of tungsten, the α line being a doublet. They are more clearly shown in figure 2, which was taken with narrower slits and greater distance of photographic

FIG. 2

plate from crystal, so that the doublet is clearly resolved. In this photograph the second order was given an extra exposure, which accounts for the apparent band in the middle. In the photograph of figure 1, all parts were given the same exposure. The wave-lengths of the lines are about 6% less than would be required by Mosely's formula. Slight deviations from the formula have already been noted for the K lines measured by Mosely and Malmer. All these lines, including tungsten, can, however, be correctly represented by the empirical formulae

$$\nu_\alpha = 1.64 \times 10^{15} N^{2.10} \text{ for the } \alpha \text{ lines,}$$

and

$$\nu_\beta = 1.56 \times 10^{15} N^{2.15} \text{ for the } \beta \text{ lines,}$$

where ν is the frequency and N the atomic number. These formulae, while they have no theoretical significance, may be useful for interpolation.

The bands which terminate on the long wave-length side at Ag_1 and Ag_2 are due to the silver in the photographic plate and are produced

by the reflection, in the first and second order respectively, of those wave-lengths which are capable of stimulating the characteristic radiation of silver, that is, those which are shorter than the gamma line of silver. The edge of the band therefore marks the position of the γ line of silver, 0.488 A.U. In the same way the band terminated at Br is

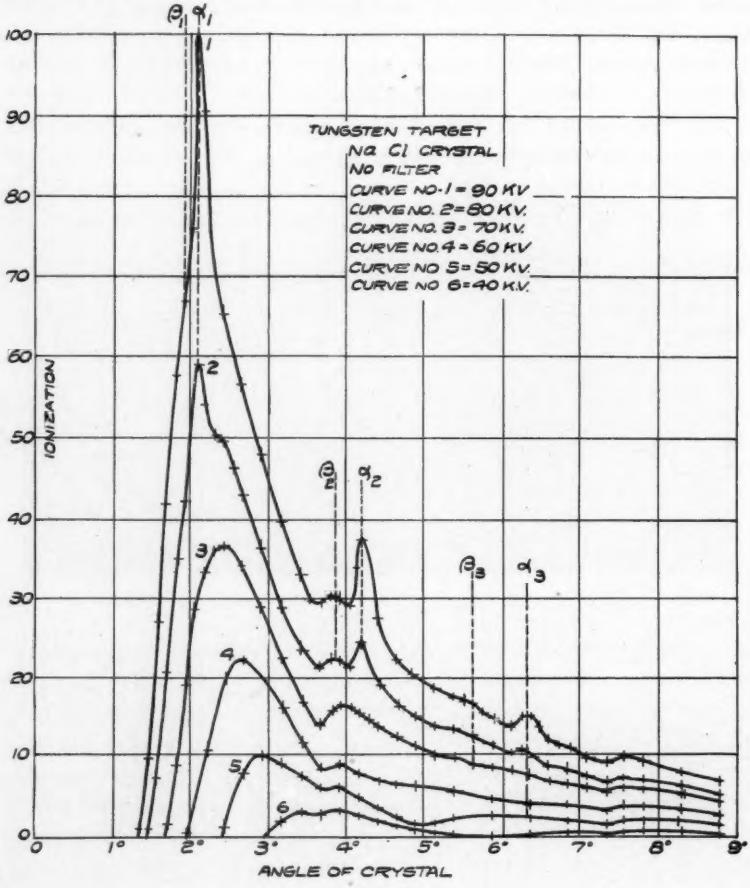

TUNGSTEN TARGET
No. Cl CRYSTAL
NO FILTER
CURVE NO. 1 = 90 KV
CURVE NO. 2 = 80 KV.
CURVE NO. 3 = 70 KV.
CURVE NO. 4 = 60 KV
CURVE NO 5 = 50 KV.
CURVE NO 6 = 40 KV.
IONIZATION
ANGLE OF CRYSTAL

FIG. 3

due to the bromine in the photographic plate, and marks the position of the γ line of bromine.

The remaining nine lines, a ..... k, belong to the L spectrum of tungsten, and agree with those found by Barnes,[5] with the inclusion of two faint lines not observed by Barnes. The line marked h appears to be a doublet. Gorton's[8] values are all about 2% smaller.

The position of the K lines, and their relation to the general radiation at different voltages, has been studied by means of the ionization chamber also. Figure 3 shows the ionization current as a function of the angle of incidence of the rays on the crystal, for five different voltages, and figure 4 a part of the same in the second order. The position of the K lines in the first, second and third order respectively, is shown by the dotted lines marked α_1, β_1, α_2, β_2, etc. There is no trace of the

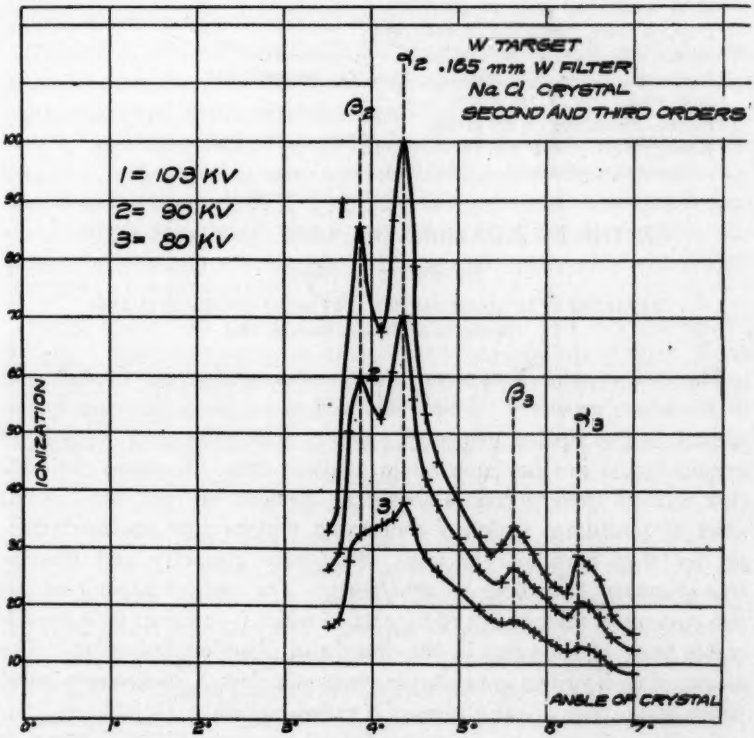


FIG. 4

lines at 70,000 volts, but at 80,000 they are clearly visible, and increase in intensity as the voltage increases. It is probable that the lowest voltage at which the lines appear is the 'quantum' voltage, i.e., that given by Duane and Hunt's equation, for the γ line, about 70,000 volts for tungsten. This would be in harmony with the mechanism of radiation suggested by Kossel[9] and has already been found by Webster[10] to hold for rhodium. It may be mentioned that the quantum relations

established by Kossel between the frequencies of the K and L lines hold true for the tungsten lines.

A more detailed account of these and other experiments on the tungsten spectrum, including the distribution of energy in the continuous spectrum, will be published shortly in the *Physical Review*.

[1] Mosely, *Phil. Mag.*, **26**, 210 and 1024 (1913); **27**, 710 (1914).
[2] I. Malmer, *Phil. Mag.*, **28**, 787 (1914).
[3] W. H. Bragg, *Phil. Mag.*, **29**, 407 (1915).
[4] Duane & Hunt, *Physic. Rev.*, **6**, 166 (1915).
[5] Barnes, *Phil. Mag.*, **30**, 368 (1915).
[6] Rutherford, Barnes and Richardson, *Phil. Mag.*, **30**, 339 (1915).
[7] Hull, *Physic. Rev.*, **7**, 156 (1916).
[8] Gorton, *Physic. Rev.*, **7**, 203 (1916).
[9] Kossel, *Ber. D. Physik. Ges.* **16**, 953 (1914).
[10] Webster, these Proceedings, **2**, 90 (1916).

ON THE FOUNDATIONS OF PLANE ANALYSIS SITUS

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The notions point, line, plane, order, and congruence are fundamental in Euclidean geometry. Point, line and order (on a line) are fundamental in descriptive geometry. Point, limit-point and regions (of certain types) are fundamental in analysis situs. It seems desirable that each of these doctrines should be founded on (developed from) a set of postulates (axioms) concerning notions that are fundamental for that particular doctrine. Euclidean geometry and descriptive geometry have been so developed.[1] The present paper contains two systems of axioms, Σ_2 and Σ_3, each of which is sufficient for a considerable body of theorems in the domain of plane analysis situs. The axioms of each system are stated in terms of a class, S, of elements called *points* and a class of sub-classes of S called *regions*.

On the basis of Σ_2, *the existence of simple continuous arcs*[2] *is proved as a theorem.*

The system Σ_2 contains an axiom (Axiom 1) which postulates the existence of a countable sequence of regions containing a set of subsequences that close down in a specified way on the points of space. Among other things this axiom implies that the set of all points is separable.[3]

The system Σ_3 is obtained from Σ_2 by replacing Axioms 1, 2, and 4 by three other axioms, Axioms 1′, 2′, and 4′ respectively. Here Axiom 1′ postulates the existence, *for each point* P, of a countable sequence of regions

that closes down on P, Axiom 2′ postulates that every two points of a region are the extremities of at least one simple continuous arc that lies in that region, and Axiom 4′ postulates that every region plus its boundary possesses the Heine-Borel property.

An open curve is defined as a closed, connected[4] set of points K such that if P is any point of K then $K-P$ is the sum of two mutually exclusive connected point-sets neither of which contains a limit point[4] of the other one. It is proved that every open curve is unbounded[4] and divides the set of all remaining points into two domains.

Though Σ_3 is a sufficient basis for a very considerable part of plane analysis situs, nevertheless there exist spaces that satisfy Σ_3 but are neither metrical, descriptive[5] nor separable.

It is interesting that no space that satisfies Σ_3 can be potentially descriptive without being also separable and potentially metrical. Indeed if to Σ_3 there be added the axiom that there exists a system of open curves such that through every two points there is one and only one curve of this system, the resulting set of axioms is categorical with respect to *point* and *limit point of a point-set.*

Every space that satisfies Σ_2 satisfies also Σ_3, but not conversely. In every space satisfying Σ_2 there exists infinitely many open curves through any two given points. I have not however determined whether every such space is descriptive.

Definitions.—A point P is said to be a *limit point* of a point-set M if, and only if, every region that contains P contains at least one point of M distinct from P. The *boundary* of a point-set M is the set of all points $[X]$ such that every region that contains X contains at least one point of M and at least one point that does not belong to M. If M is a set of points, M' denotes the point-set composed of M plus its boundary. A set of points K is said to be *bounded* if there exists a finite set of regions $R_1, R_2, R_3 \ldots R_n$ such that K is a sub-set of $(R_1+R_2+R_3+\ldots+R_n)'$. If R is a region the point-set $S-R'$ is called the *exterior* of R. A set of points is said to be *connected* if however it be divided into two mutually exclusive sub-sets, one of them contains a limit point of the other one.

The System Σ_2.—Axiom 1. *There exists an infinite sequence of regions,* $K_1, K_2, K_3, \ldots$ such that (1) if m is an integer and P is a point, there exists an integer n greater than m, such that K_n contains P, (2) if P and $\bar{P}$ are distinct points of a region R, then there exists an integer δ *such that if* $n>\delta$ *and* K_n *contains* P *then* K_n' *is a subset of* $R-\bar{P}$.

Axiom 2. *Every region is a connected set of points.*

Axiom 3. *If* R *is a region,* $S-R'$ *is a connected set of points.*

Axiom 4.[6] *Every infinite set of points lying in a region has at least one limit point.*

Axiom 5. *There exists an infinite set of points that has no limit point.*

Axiom 6.′ *If R is a region and AB is an arc such that $AB-A$ is a subset of R then $(R+A)-AB$ is a connected set of points.*

Axiom 7′. *Every boundary point of a region is a limit point of the exterior of that region.*

Axiom 8. *Every simple closed curve is the boundary of at least one region.*

The System Σ_3.—The system Σ_3 is composed of Axioms 1,′ 2,′ 3, 4′, 5, 6,′ 7,′ and 8, where Axioms 1′, 2′, and 4′ are as follows:

Axiom 1′. *If P is a point, there exists an infinite sequence of regions, $R_1, R_2, R_3, \ldots$ such that* (1) *P is the only point they have in common,* (2) *for every n, R_{n+1} is a proper subset of R_n,* (3) *if R is a region containing P then there exists n such that R_n'* is a subset of R.

Axiom 2′. *If A and B are two distinct points of a region R then there exists, in R, at least one simple continuous arc from A to B.*

Axion 4′. *If R is a region, R' possesses the Heine-Borel property.*

An example of a system satisfying Σ_2 is obtained if in ordinary Euclidean space of two dimensions, the term region is applied to every bounded connected set of points M, of connected exterior, such that every point of M is in the interior of some triangle that lies wholly in M.

Details (including a third system of axioms, the system Σ_1) will appear in *Transactions of the American Mathematical Society*, probably in April, 1916.

[1] Cf., among others, D. Hilbert, *The Foundations of Geometry* (translation by E. J. Townsend, Chicago, 1902) and O. Veblen, A system of axioms for geometry, *Trans. Amer. Math. Soc.*, **5**, 343 (1904).

[2] Lennes defines a continuous simple arc connecting two points A and B, $A \neq B$, as a bounded, closed, connected set of points $[A]$ containing A and B such that no proper connected subset of $[A]$ contains A and B. Cf. N. J. Lennes, *Amer. J. Math.*, **33**, 308 (1911).

[3] A set of points M is said to be *separable* if it contains a countable subset K such that every point of M is a limit point of K. See M. Fréchet, Sur quelques points du calcul fonctionnel, *Palermo, Rend. Circ. Mat.*, **22**, 6 (1906).

[4] For definitions of these terms see below.

[5] A space S is said to be *descriptive*, or *potentially descriptive*, if it contains a system of open curves such that through every two points of S there is one and only one curve of this system.

[6] In view of a result due to F. Hausdorff, it is clear that in the presence of the other axioms of Σ_2, Axiom 4 is equivalent to the axiom that if R is a region then R' possesses the Heine-Borel property. See F. Hausdorff, Grundzüge der Mengenlehre, Veit and Comp., Leipzig, 1914.

A GENERAL THEORY OF SURFACES

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Introduction.—Although the differential geometry of a k-dimensional spread or variety V_k, embedded in an n-dimensional space S_n, has received considerable attention, in particular in the case $k = n-1$, the theory of surfaces V_2 in the general Euclidean hyperspace S_n has been treated extensively by only three authors, Kommerell, Levi, and Segre,[1] of whom the last was interested in projective properties, the first two in ordinary metric relations. The theory of V_2 in S_n does, however, offer points of contact with elementary differential geometry which are fully as illuminating as those developed in the case of V_{n-1} in S_n. We have, therefore, undertaken to provide a general theory of surfaces which shall be independent of the number of dimensions of the containing space.

The first thing to be determined in entering on such an extended study is the method of attack. The available methods were four: (1) The ordinary elementary method of starting with the finite equations of the surface and trying to generalize well-known geometric properties. This was followed by Kommerell. (2) The more advanced method of Levi, which depends upon the finite equations only for calculating certain invariants I of rigid motion and further invariants (covariants) J of transformations of parameters upon the surface. (3) The vectorial method of Gibbs and others which bases the work upon the properties of the linear vector function expressing the relation between an infinitesimal displacement in the surface and the infinitesimal change in the unit normal $(n-2)$-space or the unit tangent plane. (4) The method of Ricci's absolute differential calculus, or Maschke's symbolic invariantive method, which develops the theory from the point of view of the fundamental quadratic differential forms defining the surface.

We adopted the last method because of the possibility of sharply differentiating (1) those properties of surfaces which depend on the first fundamental form, and belong to the surface as any one of the infinite class of mutually applicable surfaces, from (2) the properties which follow from the first and second fundamental forms taken together and which belong to the surface as a rigid surface. The adoption of Ricci's rather than Maschke's form of analysis was further dictated by the fact that Ricci[2] had developed the ordinary theory of surfaces consistently by his method, and all his work upon the first part (1), holding without

change in any number of dimensions, could stand without repetition. Our own work therefore need only begin with the study of the second fundamental form. As, however, Ricci's absolute differential calculus is neither well-known nor published in particularly accessible form, it seemed best to prefix to our work an explanation of that calculus.

The second fundamental form.—If $\mathbf{y}\,(u, v)$ be the vector which determines the points on the surface, the second fundamental form is found to be a vector form

$$\Psi = \mathbf{y}_{11}dx_1^2 + 2\,\mathbf{y}_{12}\,dx_1dx_2 + \mathbf{y}_{22}dx_2^2 = \Sigma_{rs}\,\mathbf{y}_{rs}\,dx_rdx_s,$$

where $\mathbf{y}_{11}$, $\mathbf{y}_{12}$, $\mathbf{y}_{22}$ are the second covariant derivatives of $\mathbf{y}$ with respect to u and v and are vectors normal to the surface. If ds denote a differential of arc and

$$\lambda^{(r)} = dx_r/ds, \qquad \lambda'^{(r)} = dx_r/ds', \qquad r = 1, 2,$$

be the differential equations of a system of curves λ on the surface and of their orthogonal trajectories, the three vectors α, β, μ,

$$\alpha = \Sigma_{rs}\,\lambda^{(r)}\,\lambda^{(s)}\,\mathbf{y}_{rs}, \qquad \beta = \Sigma_{rs}\,\lambda'^{(r)}\,\lambda'^{(s)}\,\mathbf{y}_{rs},$$
$$\mu = \Sigma_{rs}\,\lambda'^{(r)}\,\lambda^{(s)}\,\mathbf{y}_{rs} = \Sigma_{rs}\,\lambda^{(r)}\,\lambda'^{(s)}\,\mathbf{y}_{rs},$$

are invariant of the parameter system on the surface and are associated with a direction λ. Further

$$\mathbf{y}_{rs} = \alpha\,\lambda_r\,\lambda_s + \mu\,(\lambda_r\,\lambda' + \lambda'_r\,\lambda_s) + \beta\,\lambda'\,\lambda',$$

where λ_r is the dual or reciprocal system to $\lambda^{(r)}$ defined by $\lambda_r = \Sigma_s a_{rs}\,\lambda^{(s)}$ if $ds^2 = \Sigma_{rs}\;a_{rs}\,dx_r\,dx_s$ be the first fundamental form.

The geometric interpretation of α is the normal curvature of the surface in the direction λ (i.e., the curvature of the geodesic tangent to λ); of β, the same for the perpendicular direction; of μ, the rate of change of a unit vector drawn in the surface perpendicular to the geodesic tangent to λ. The Gaussian or total curvature G is $G = \alpha\cdot\beta - \mu^2$, the difference of the scalar product of α and β and the square of μ; this is independent of the direction λ and is one of the prime invariants of the surface. There is a vector $\mathbf{h}$ defined by the equation

$$2\mathbf{h} = \alpha + \beta = \Sigma_{rs}a^{(rs)}\,\mathbf{y}_{rs}, \qquad a^{(rs)} = (-1)^{r+s}a_{rs}/\,|\,a_{rs}\,|,$$

which is also independent of the direction λ and which we call the mean (vector) curvature of the surface at the point considered; the magnitude of $\mathbf{h}$ is another of the prime invariants of the surface.

If $\mathbf{M}$ be a unit (vector) tangent plane to the surface, and $d\mathbf{M}$ its differential, the second fundamental (vector) form Ψ may be written

$\Psi = -(d\mathbf{y}\cdot\mathbf{M})\cdot d\mathbf{M}$, where the dot denotes the inner product,[2] in complete analogy with the expression $-d\mathbf{y}\cdot d\mathbf{n}$, in terms of the unit normal $\mathbf{n}$, in ordinary surface theory. Moreover, the square of $d\mathbf{M}$ is $(d\mathbf{M})^2 = -Gds^2 + 2\mathbf{h}\cdot\Psi$, and thus gives what may be called the third fundamental form. (In the three-dimensional case, this is associated with the spherical representation.)

The indicatrix.—If $\delta = \frac{1}{2}(\alpha-\beta)$ and if accents be used to denote quantities corresponding to a direction at an angle θ to λ, we find

$$\mu' = \mu\cos 2\theta - \delta\sin 2\theta, \qquad \delta' = \delta\cos 2\theta + \mu\sin 2\theta.$$

This means that when we turn about a point of the surface the vectors μ', δ' describe an ellipse (with center at the extremity of the mean curvature $\mathbf{h}$) of which any two positions of μ', δ' are conjugate radii, the eccentric angle between μ', μ or δ', δ being twice the angle θ. This ellipse we call the indicatrix.[4] The vectors α', β' originating at the surface-point O and terminating in the indicatrix describe a cone of normals, Cone I.

The indicatrix is determinative of a large number of properties connected with the curvature of a surface at a point. As the surface is two-dimensional and the indicatrix with the surface-point O determines a normal three-dimensional space, the properties of curvature are as general for a surface V_2 in a five-dimensional space S_5 as in S_n, and the further developments may be given in the assumption that $n = 5$.

Consecutive normal plane three-spaces N_3 intersect in a line. These normal lines all pass through a point O' lying upon the perpendicular from O, to the plane of the indicatrix and generate a quadric cone, Cone II. The relation between Cones I and II is reciprocal in the sense that each element of either is perpendicular to some tangent plane of the other. Hence if Φ be the linear vector function which occurs in the equation $\mathbf{r}.\Phi.\mathbf{r} = 0$ of Cone II, referred to its vertex, the equation of Cone I, referred to its vertex, is $\mathbf{r}\cdot\Phi^{-1}\cdot\mathbf{r} = 0$.

The function Φ may be written as $\Phi = (\mathbf{hh} - \mu\mu - \delta\delta)/a$, where $a = |a_{rs}|$, in terms of the mean curvature $\mathbf{h}$ and a pair of conjugate radii of the indicatrix. Φ is the self conjugate part of the function

$$\Omega = \begin{vmatrix} \mathbf{y}_{11} & \mathbf{y}_{12} \\ \mathbf{y}_{21} & \mathbf{y}_{22} \end{vmatrix} = \mathbf{y}_{11}\,\mathbf{y}_{22} - \mathbf{y}_{12}\,\mathbf{y}_{21}$$

formed as the determinant (with vector elements) of the second form Ψ. We have

$$\Phi = \tfrac{1}{2}\,\mathbf{y}_{11}\,\mathbf{y}_{22} + \tfrac{1}{2}\,\mathbf{y}_{22}\,\mathbf{y}_{11} - \mathbf{y}_{12}\,\mathbf{y}_{21},$$

and the total curvature G is

$$G = \alpha \cdot \beta - \mu^2 = \Phi_2/a = \Omega_2/a,$$

the ratio of the first scalar invariant of Φ, or of the determinant Ω of the second form Ψ, to the determinant a of the first form.

If the surface V_2 in S_5 be projected successively upon the three spaces S_3 determined by the tangent plane and each of three mutually perpendicular normals issuing from O into the normal space N_3, the three projections have the (vector) sum of their mean curvatures and the (scalar) sum of their total curvatures equal respectively to the mean curvature and total curvature of the given surface at O. There is therefore a cone, Cone III, of normals such that if any element of the cone be chosen as one of the three mutually perpendicular normals, the other two may be chosen in such a way (upon Cone II) that the total curvature of the three projections are respectively G, 0, and 0. The equation of this cone is $\mathbf{r} \,.\, (\Phi_2 I - \Phi) \,.\, \mathbf{r} = 0$, where I is the idemfactor.

Types of surfaces.—For minimal surface $\mathbf{h} = 0$, and the indicatrix everywhere reduces to an ellipse in some normal plane and with its center at the surface-point O.

For the surface formed by the tangents to a twisted curve, the indicatrix reduces to a segment of a line (described twice) reaching from O to the extremity of the vector $2\,\mathbf{h}$. Such a surface is developable, and all ruled developables are of this type.

For any ruled surface the indicatrix lies in a plane passing through the surface-point O, and the ellipse itself passes through O. The total curvature of any real ruled surface (other than developable) is negative.

For a surface of revolution, which is formed by revolving a twisted curve parallel to a plane, the indicatrix reduces to a linear segment (described twice) centered at the extremity of $\mathbf{h}$. There is a large variety of developable surfaces (not ruled) of revolution, of which the simplest is perhaps that obtained by revolving the circular helix parallel to a plane containing its axis and a line perpendicular to the three-space in which the helix lies.

For a developable (non-ruled), the indicatrix is tangent to three mutually perpendicular planes through O, or, if $n = 4$, to two perpendicular lines through O. A developable is not invariant in type under a projective transformation.

The class of surfaces where the indicatrix at each point lies in a plane with O is invariant under projective transformations, and these surfaces have upon them the characteristic lines of Segre.

A special type of surface, which have the indicatrix everywhere a

linear segment noncollinear with $\mathbf{h}$, is a simple generalization of ordinary surfaces ($n = 3$) in that by a proper choice of parametric curves the first and second fundamental forms reduce simultaneously to sums of squares. The property is not invariant under a projective transformation.

Lines upon surfaces.—Kommerell defines as principal directions those for which the normal curvature is a maximum or minimum, and subsequent authors have followed him. This gives four directions at each point, and they are not simply related to one another. We have chosen to give as a definition of principal directions those for which $\mathbf{h}$ and μ are perpendicular. There are two such directions at each point, they are orthogonal and have the properties that the differential tangent planes in the two directions are perpendicular and the values of the rate of change $d\mathbf{M}/ds$ of the tangent plane is numerically a maximum or minimum.

Kommerell defines asymptotic lines, and Levi follows him after restricting the definition to a special case. The definition is not satisfactory, and we define asymptotic lines as those for which $\mathbf{h}$ and α are perpendicular. The rate of turning of the tangent plane along these directions is the square root of the negative of the total curvature. The differential equations of the lines are $\mathbf{h} \cdot \Psi = 0$. The lines are bisected by the principal directions.

Our definitions for principal directions and asymptotic lines become illusory for minimum surfaces.

Segre's characteristics exist when the indicatrix lies in a plane with the surface point O and are then those directions which make α tangent to the indicatrix. The asymptotic directions divide the characteristic directions harmonically.

Development of surfaces.—In the neighborhood of a point a surface may usually be developed in either of two standard forms.

$$\begin{aligned} & z_1 = \tfrac{1}{2} h(x^2 + y^2) + e(x^2 - y^2), && z_2 = \tfrac{1}{2} f(x_2 - y^2), \\ & z_3 = \tfrac{1}{2} A(x^2 - y^2) + 2\,Bxy, && z_i = 0,\ i > 3, \\ \text{or} \quad & z_2 = \tfrac{1}{2}(Ax^2 + 2\,Bxy + Cy^2), && z_3 = \tfrac{1}{2} Dx^2,\ z_3 = \tfrac{1}{2} Ey^2. \end{aligned}$$

For the first form $\mathbf{h}$ lies along the axis of z_1, the plane of the indicatrix is parallel to the axis of z_3, and the axis of x and y are along the principal directions. A hyperplane tangent to the surface at O will cut the surface in real, imaginary, or coincident directions according as it cuts the indicatrix in real, imaginary, or coincident points. The second standard form of the surface has the property that the two perpendicular hyper-

planes $z_2 = 0$ and $z_3 = 0$ cut the surface in orthogonal coincident directions.

Particular interest attaches to the tangent hyperplane perpendicular to the mean curvature h. This cuts the surface in the asymptotic directions and the axes of the (degenerate) conic made up of these two directions are the principal directions. The intersection of the surface and this hyperplane has therefore the fundamental properties of the Dupin indicatrix.

For the proof of the geometric results here stated and for the proofs and statements of a large number of others, many of which are entirely new, some only new statements of the results of Levi, Kommerell, or Segre, reference must be made to our complete memoir 'Differential Geometry of Two-dimensional Surfaces in Hyperspace' which will be published in the *Proceedings of the American Academy*, Boston.

[1] Kommerell, Die Krümmung der Zweidimensionalen Gebilde in ebenen Raum von vier Dimensionen, *Dissertation*, Tübingen, 1897, 53 pp; E. E. Levi, Saggio sulla Theoria delle Superficie a due Dimenzioni immersi in un Iperspazio, *Pisa, Ann. R. Scu. Norm*, **10**, 99 pp; C. Segre, Su una Classe di Superficie degl' iperspazi, *Torino, Att. R. Acc. Sci.*, **42**, 1047–1079 (1907).

[2] Ricci, *Lezioni sulla Theoria delle Superficie*, Padova, Drucker, 1898. (Lithographed, edition exhausted.)

[3] The vector analysis used is a modification of the Grassmannian system; see Lewis, *Proc. Amer. Acad. Arts Sci.*, **46**, 165–181 (1910), and Wilson and Lewis, *Ibid.*, **48**, 389–507 (1912).

[4] The corresponding indicatrix for V_2 in S_3 is not Dupin's but the range of points upon the normal described (twice) by the normal curvature vector α.

DYNAMICAL STABILITY OF AEROPLANES

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The first rational theory of the dynamical stability of aeroplanes is due to Bryan[1] whose work was extended and applied by Bairstow[2] with wind tunnel tests on models.

The utility of such tests in predicting the aerodynamical properties of a full size aeroplane is now well understood and the validity of this application has been repeatedly demonstrated. The late E. T. Busk of the Physical Staff of the Royal Aircraft Factory, England, applied Bairstow's model tests to the design of an aeroplane and recently succeeding in perfecting an inherently stable machine which could be flown 'hands off.' Neither the details of Busk's experiments nor of the type of aeroplane developed by him have been disclosed by the British War Office.

My present investigation of two different types of aeroplanes, one a standard military tractor with no claims to inherent stability in flight, the other a machine designed to possess some degree of inherent stability while departing as little as possible from standard practice as exemplified by the other, has for its object:

(*a*) To determine the aerodynamical constants for the two aeroplanes by means of model tests in the wind tunnel.[3]

(*b*) To apply the aerodynamical constants so found in the dynamical equations of motion for the full scale aeroplanes in free flight and to examine the stability of the motion.

(*c*) To compare the stability, both lateral and longitudinal of the two chosen types of aeroplane, with a view to tracing to the individual parts of each machine their effects upon the motion.

(*d*) To attempt to formulate general qualitative conclusions which may assist constructors of aeroplanes to avoid instability or to provide any desired degree of stability.

(*e*) To throw light upon the general problem of inherent dynamical stability.

Stability distinguished as statical or dynamical.—An aeroplane in horizontal flight in still air must be driven at such speed and kept at such an inclination of wings to wind that the weight is just sustained. When in this normal attitude the aeroplane, if properly balanced, is in equilibrium. In a statical sense, this equilibrium is stable if righting moments are called into play tending to return the aeroplane to its normal attitude if by any cause it is deviated thereform. In general an aeroplane, if stable in a statical sense, will, when given an initial deviation, take up an oscillation which either may be damped out or may increase in amplitude. The aeroplane is dynamically stable if, and only if, these oscillations die out as time goes on, leaving the aeroplane in its original normal attitude. It is clear that statical stability must first be provided before the dynamical stability of a design can be examined.

The righting moments which tend to restore the aeroplane to its normal attitude are a measure of statical stability. If the statical stability is great, the period of the oscillations will be short and the motion violent, whereas an aeroplane should have a gentle motion of slow period heavily damped. An ocean liner of too great stiffness (large metacentric height) is not suitable for passenger or cattle carrying service; it is preferable in ship-design to give only enough metacentric height to insure that the vessel in never unstable and to damp the roll by generous bilge keels. In a similar manner the theory indicates that, in aeroplane-design, it is preferable to give just enough statical stability to insure that

the aeroplane is never unstable and to make the oscillation dynamically stable by the use of generous damping surfaces and large wings.

The investigation is most conveniently discussed in two parts, the first dealing with the 'longitudinal' motion involving pitching of the aeroplane and rising and sinking of its center of gravity combined with change of forward speed, and the second with the 'lateral' motion, which involves side-slipping, or skidding, combined with rolling and yawing to the right and left.

Longitudinal motion.—From the dynamical equations of motion for the full scale aeroplane written down with the aerodynamical coefficients determined by tests on the models, the small oscillations about the equilibrium position are determined by three simultaneous linear differential equations with constant coefficients, of which the solution shows that the motion may be considered as composed of two oscillations one of a period of the order of 2 seconds damped to half amplitude in 0.1 seconds, the other of period from 10 to 30 seconds not very strongly damped. The short oscillation appears never to be of importance in ordinary aeroplanes, but the long oscillation, being only moderately damped, may cause trouble, especially for an aeroplane that flies with a large angle of incidence for the wings.

The calculation of the period and damping of the long oscillation was repeated for several speeds from highest to lowest corresponding to small and large angles of incidence with the results shown in the following table:

Aeroplane*	S	S	S	S	U	U	U	U
Velocity, miles-hour	76.9	53.4	44.6	36.9	79	51.8	47	44.2
Incidence of wings	0°	30°	6°	12°	1°	7°	10°	14°
Period, seconds	34.7	17.6	15.8	10.56	34.0	16.7	13.7	11.5
Damp 50 per cent, seconds	8.1	11.0	13.1	—	11.0	17.7	63.0	—
Double amplitude, seconds	—	—	—	24.7	—	—	—	24.7

* The letters S and U represent respectively the machine designed for inherent stability and a standard military tractor.

Instability at low speed.—It appears that the period becomes much more rapid at low speed, that at some critical speed the damping vanishes, and below this speed both aeroplanes become frankly unstable. This instability at extreme low speed is common to all aeroplanes and the only advantage of our 'stable' aeroplane S is that its longitudinal motion is stable down to about 40 miles per hour while aeroplane U is stable only down to about 47 miles per hour.

A study of the relative magnitudes of the coefficients for these typical aeroplanes leads to the conclusion that longitudinal instability at low

speed is due, first, to the decrease in damping of the tail surfaces on account of the low speed and, secondly, to the decrease in rate of change of lifting force with change in attitude for high angles of incidence. The latter has a predominating effect on the damping of the long oscillation. Consequently, if an aeroplane is to be stable and land at a relatively slow speed, it must not operate at too great an angle of incidence. To sustain its weight it should therefore have a comparatively large wing area. The principal difference between aeroplanes S and U is that the former supports a weight of 3.55 pounds per square foot of wing area and the latter 5.2 pounds per square foot.

The following recommendations are made for an aeroplane to have its longitudinal motion damped at lower speeds than is usual in practice: (1) Provided large horizontal surfaces of long arm for damping the pitching. (2) Provide wings of such area that the slow speed does not require a great angle of incidence. Roughly the safe slow speed should not require more than 80 per cent of the maximum lift of the wings. (3) Keep the longitudinal radius of gyration small by concentrating the principal weights.

Slowness in pitching.—It may be imagined that a dynamically stable aeroplane of rapid period might be so violent in its motion that the pilot would be shaken about to such an extent as to be hindered in the performance of his military duties of observation, gun-fire, or bomb dropping. It appears that the expression representing the period of the long oscillation contains certain predominating coefficients, and a consideration of their magnitude leads to the following conclusions: The natural period of pitching is increased by: (1) High speed of flight, (2) Large damping surfaces on the tail, (3) Small angle of incidence, (4) Small righting moments.

Lateral motion.—After measuring the aerodynamical coefficients, and the radii of gyration in roll and yaw, the dynamical equations for the asymmetrical or lateral motion may be set down. For small oscillations these reduce, as in the longitudinal case, to three linear differential equations with constant coefficients. The determinant formed from the coefficients may be factored by use of approximate methods and the motion may be compounded from that represented by each of three factors.

Spiral Dive.—The first factor may correspond either to a damped or to an amplified motion. At high speeds model S shows a subsidence damped to half amplitude in 10.4 seconds. At lower speeds this damping diminishes and at 37 miles per hour the motion becomes a divergence which doubles in amplitude in 7.2 seconds. Aeroplane U is spirally unstable at high speeds. Examination of the preponderating terms in the

expression representing the motion shows that the aeroplane starts off on a spiral dive.

A simple relation may be obtained involving four of the aerodynamical coefficients which, if positive, insures that spiral instability of this kind is not present. It appears that spiral instability is caused by too much fin surface to the rear or to too large a rudder, and by not enough fin surface above the center of gravity. A proper adjustment is easily obtained without sacrifice of desirable flying properties. Aeroplane S has a small rudder and wing-tips raised about 1°.6; aeroplane U has no rise to wing-tips nor vertical surface above the center of gravity and has a very deep body giving the effect of a rear vertical fin. These differences in design account for the respective stability and instability of the two machines.

Rolling.—The second factor in the equation of motion represents a rolling of the aeroplane which is so heavily damped by the wide spreading wings as to be ordinarily of no consequence. In the extreme case of a 'stalled' aeroplane, the damping of the roll vanishes because the downward moving wing has no more lift than the other. Here we may expect trouble, and frequent accidents to stalled aeroplanes indicate that the pilot's lateral control by ailerons also becomes operative.

Dutch Roll.—The third element in the motion is a yawing to right and left, combined with rolling. The motion is oscillatory of period from 5 to 12 seconds, which may or may not be damped. The analogy to the 'Dutch Roll' or 'Outer Edge' in ice-skating is obvious. If the skater lean too far out on his swings he may fall, and in the same manner if the aeroplane bank too much a slight puff of wind may capsize it.

The motion of the Dutch Roll is stable provided there be sufficient vertical fin surface on the tail and not too much fin surface above the center of gravity. These requirements conflict with those previously stated for spiral stability and a compromise must be made. Over-correction of spiral instability may produce instability in the Dutch Roll and vice versa. Fortunately, the damping of rolling by the wings is helpful in both cases, and it appears possible to obtain that nice adjustment of surfaces which will render both motions stable.

Model S was stable in the Dutch Roll at all speeds, having a period from 6 to 12 seconds, and the initial amplitude damped 50 per cent in from 1.5 to 6 seconds. Model U was stable in this respect except at low speed when it showed a period of 6 seconds and the initial amplitude was doubled in 8 seconds.

The following table summarizes the results obtained for the lateral motion.

Aeroplane	S	S	S	U	U
Rise of wings	1°63	—	—	0	—
Angle of incidence	0°	6°	12°	1°	15°5
Velocity, miles	76.9	44.6	36.9	78.9	43.6
Spiral motion					
Damp 50 per cent, seconds	10.4	2.7	—	—	3.3
Double, seconds	—	—	7.2	28.0	—
Dutch roll					
Period, seconds	5.9	10.7	12.0	5.2	5.7
Damp, 50 per cent seconds	1.4	1.3	6.0	1.8	—
Double, seconds	—	—	—	—	7.7

General Conclusions.—It is believed that the majority of modern aeroplanes are spirally unstable but stable in the Dutch Roll. Furthermore it appears to be a simple matter so to adjust surfaces that any aeroplane can be made completely stable without sacrifice in speed or climb. At extreme low speed an aeroplane must be unstable in its longitudinal motion but need not be unstable laterally.

The degree of stability to provide in a given case cannot be determined from mechanical considerations alone. For example, the comfort of the pilot must be a first consideration and for this reason the righting moments giving statical stability should be small; the period of the aeroplane can then be made relatively slow, and if the damping is adequate, the free oscillations will be stable.

The theory is applied here only to flight in still air. Obviously the air is never still, and the aeroplane must finally be judged from its behavior in gusts. An inherently stable aeroplane tends to preserve its normal attitude with relation to the relative wind, and if the velocity and direction of the relative wind change in an irregular manner, the stable aeroplane will tend to follow. The result will be to force on the aeroplane a motion which will be more violent the greater the statical stability. Consequently in rough air an aeroplane very stable statically is unsuitable as a gun platform and for many other military purposes.

Considerations of theory indicate that a slight degree of statical stability combined with the maximum of damping give an aeroplane slow periods of oscillation and a dynamically stable motion, with little ill effect upon performance or controllability.[4]

[1] G. H. Bryan, *Stability in Aviation*, Macmillan, 1910.

[2] L. Bairstow, *Technical Report of the Advisory Committee for Aeronautics*, London, 1912–13.

[3] A description of the wind tunnel and the results of some experiments therein may be found in *Smithsonian Inst., Misc. Coll.*, **62**, No. 4, 1–92 (1916).

[4] Full details of this investigation will be offered for publication in a forthcoming number of *Smithsonian Misc. Coll.*

CLIFT ISLANDS IN THE CORAL SEAS

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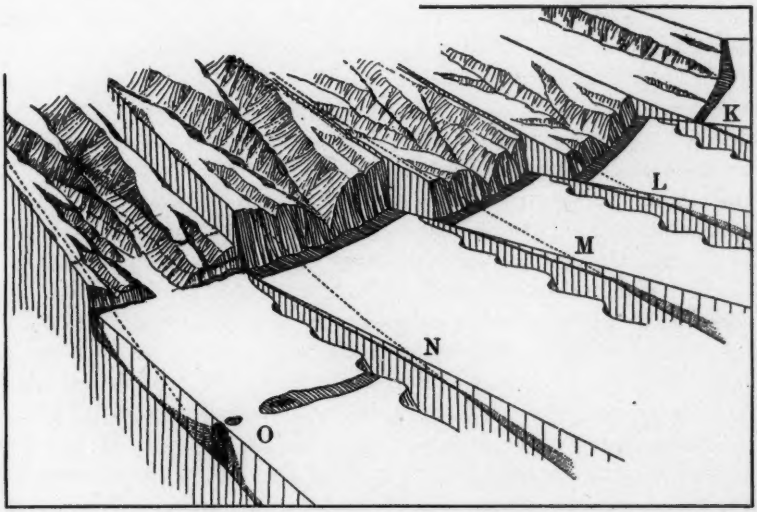
The great majority of reef-encircled islands in the Coral Seas are of maturely dissected form, with slopes of moderate declivity that descend to an indented shoreline, where the tapering spurs end in salient points and the widening valleys open into branching bays, thus proclaiming that the islands have suffered partial submergence since they were dissected. It is generally agreed by those who accept this interpretation of insular relief in relation to shoreline pattern that the encircling barrier reefs have grown upwards from their beginning as fringing reefs during the subsidence which gave the shoreline its indented outline. Reasons for this conclusion were given in a previous article 'The Origin of Coral Reefs' in these PROCEEDINGS 1, 146–152, 1915; they have been more fully stated in 'A Shaler Memorial Study of Coral Reefs' recently published in the *American Journal of Science*, **40**, 223–271, 1915.

The object of the present article is to call attention to certain exceptional reef-encircled islands, Reunion, Tahiti, and New Caledonia, in the Coral Seas, which present clift shore lines, and to suggest an explanation by which their exceptional features may be simply accounted for. If all the islands of the Coral Seas were clift like these three, they would support either the theory of veneering barrier reefs on wave-cut platforms, or the Glacial-control theory of coral reefs. But clift islands are exceptional. Some means of explaining them must be found which will not be in discord with the theory of coral reefs that is supported by the features of non-clift islands, namely, Darwin's theory of intermittent subsidence. It will appear from what follows that the suggested means of explaining the exceptional clift islands of the Coral Seas gives unexpected support to this theory.

Imagine a young volcanic island, from which detritus is washed so abundantly into the sea that corals and the associated reef-making organisms can establish themselves if at all, only in short and discontinuous fringing-reef patches around its shore. So long as the island continues to grow by eruption, its height and perimeter will increase; its earlier fringing reef patches will be buried under newer lava flows, agglomerate beds and ash falls. The completed volcanic island will have a shoreline of simple pattern without pronounced salients and reentrants, because the materials ejected from the summit crater are spread rather equably on all sides of the cone, and thus give it a nearly circular base. The sim-

ple shoreline may be at first somewhat prograded by the deltas of the larger streams, sector K of the figure; between the deltas it may be somewhat retrograded and clift by the waves; but an abundant supply of cobbles, gravels and sands from the delta fronts will be swept along the inter-delta shores in the form of beaches that will cover the cliff-base platforms; and the finer sediments will be spread seaward on the submarine flanks of the volcanic mass. The deltas, the beaches and the off-shore sediments will form a detrital sheet, thickest and of coarsest texture on the deltas, of finest texture and for a time thinnest where it slopes into deep water; but as time passes, the fine-textured, off-shore sediments will come to exceed all other parts of the detrital sheet in thickness, for



when retrogradation begins and the deltas are cut back in the general retreat of the shoreline, sectors L, M, N, the stream-mouth deposits will be hardly thicker than the cliff-base beaches. Small patches of fringing reef may at first form on the bare unreduced ledges that here and there protrude through the cliff-base beaches; but elsewhere the materials of the detrital sheet are too commonly in movement to permit coral growth.

After volcanic eruptions have definitively ceased and the normal dissection of the cone by weather and streams and the cutting of cliffs around the simple shore by waves are further advanced, it is probable that the first formed reef patches will be overwhelmed by the increasing continuity of the detrital sheet, and if this unfavorable condition for reef

development once sets in, it is likely to continue indefinitely, as long as the island stands still. The grinding of the detritus upon itself, as it is shifted by the waves, would soon reduce it to so fine a texture that it would all be swept off the platform into deep water, leaving the platform bare; but the loss thus occasioned is continually made good by a new supply from the valleys and cliffs of the island; hence the detrital sheet is always maintained. The rasping of the detritus on the platform, especially at the time of storms, slowly wears the rock surface down to greater and greater depth, but the degradation thus caused is probably 20 or 40 or more times slower than the retrogradation of the cliffs. The 'edge' at the outer margin of the abraded platform may be somewhat accentuated and stript as it is worn down, because the slow abrasion of its rock surface may be exceeded by the more rapid abrasion of the first deposited detritus next outside of it: here corals and other reef building organisms might grow, but for the constant outward passage of detritus which would smother them. The important matters to note thus far are first, that when the series of normal changes begins around the simple shoreline of the completed island, sector K, the shore waves will at once have to deal with so abundant a supply of detritus washed down by torrents, that a continuous shoreline beach and off-shore detrital sheet will be formed at an earlier stage of shoreline development than is usually the case on the shores of strongly sloping coasts; second, that the detrital sheet, once established, will be thereafter well supplied with waste from the deepening valleys and the retreating cliffs, and will thus be maintained indefinitely, as long as the island stands still; and third, that as long as the detrital sheet exists, the growth of corals is greatly discouraged if not entirely prevented.

Now let it be supposed that when a certain stage of cliff recession is reached, as in sector N, the island subsides by a succession of starts and stops, so that the shoreline crosses the cliffs at mid-height and enters the larger valleys, as in sector O. Several significant changes will follow. The detritus from the larger valleys of the dissected cone will now be pocketed in the valley-mouth embayments; the waves, beating the face of the cliffs without the aid of the boulders and cobbles, with which they had previous attacked the cliff base so successfully, will do little work in the way of cutting the upper half of the cliffs farther back. Under these conditions corals and their associates may attach themselves to the cliff face and form a fringing reef; as soon as such a reef broadens a little, it will hold nearly all the detritus that is washed from the little hanging valleys and weathered from the non-submerged part of the cliff face: thus the detrital sheet on the abraded platform, no longer receiving a renewal

of supply from the cliffs or the streams, will be slowly shifted seaward and the rock platform will be at least in part exposed, especially along the 'edge' at its outer margin, where the heavy outside storm waves make their first attack on the bottom: here in particular, the abraded rock of the platform may be laid bare and a favorable site for coral colonies provided; here, if the subsidence has not been too great, a barrier reef may grow up, as in the side section of Sector O.

As soon as the barrier reef is fairly well established, wave work in the lagoon is greatly weakened. If the island now stand still again, the lagoon will be gradually aggraded by overwashed detritus from the reef, by organic deposits formed in the lagoon, and by outwashed detritus from the island: thus a delta-and-reef plain will in time replace the lagoon, and when this stage is approached and reached, the stream-borne volcanic detritus will be again delivered on the reef face, where it will retard the growth of corals and their associates. But if subsidence be renewed and the shore line rise to the broken line of sector O favorable conditions will be re-introduced, and the barrier reef will grow upward again. The cliffs that were cut before subsidence began will in time be completely submerged; the shoreline will then have the pattern shown by a dotted line in sector O, and this pattern will persist as long as subsidence continues, until in the almost-atoll stage the diminishing island is reduced to so small a size that it has no room for valleys; only a central summit will then remain above sea level; in the next stage this disappears, and a perfect atoll results.

Actual examples of islands and reefs should be found in various stages of the sequence of events here deduced, if the scheme by which the deductions are guided is a true counterpart of natural occurrences. Reunion in the western Indian Ocean offers confirmation for sectors K and L. As described, figured and mapped by Maillard, and especially by von Drasche, it has no embayments; part of the shoreline is clift with hanging valley mouths; much of the shoreline is delta-fronted and beached; and coral reefs form only fragmentary fringes along about one-tenth of its circuit.

Tahiti, a volcanic doublet in the Society group of the South Pacific, is clift nearly all around its figure-8 circuit. The cliffs, well described by Agassiz, are highest on the eastern or windward side where some of them rise 1000 feet above the sea; a number of the smaller valleys make mere notches in the cliff top, and the streams cascade down the cliff face; a few of the larger valley mouths are still embayed as all of them once were; hence the island has subsided and the cliff base is submerged Most of the embayments are now filled with deltas, which are indeed advancing

into the lagoon, where they form a narrow alluvial plain on which most of the population dwells. A barrier reef rises at just about such a distance off shore as to suggest that its foundation is on the edge of the abraded platform that fronts the submerged base of the cliffs; volcanic detritus is abundant in the enclosed lagoon. Tahiti thus seems to have been clift, about as in sectors L, M or N, and then to have subsided as in sector O; since then it has stood still long enough for abundant delta advance and for partial enclosure by a barrier reef, the further growth of which is now threatened at certain points by the advancing deltas.

New Caledonia is peculiar in being a non-volcanic island, and in being clift only on the northeastern side: hence certain special assumptions must be made in order to bring it under the explanatory scheme here proposed; but the necessary assumptions do not involve any events that are not made probable by what is known of the geological and physiographical history of the island.

The great majority of reef-encircled volcanic islands in the Pacific are volcanic, like Reunion and Tahiti; but they differ from these two submaturely dissected cones in being maturely or more than maturely dissected, so that hardly a trace of their initial conical slopes remains; furthermore, their spurs usually taper to non-clift points and their valleys open into broadening bays, as shown in the dotted high-level shoreline of sector O. If the reef-encircled islands of the Pacific have formerly passed through an introductory, almost reefless stage, like Reunion, and a soon-following reef-encircled stage with partly drowned cliffs and outgrowing deltas, like Tahiti, they have since then continued their subsidence as well as their dissection and their reef growth, so that at present they have no trace of the smooth slopes of their initial conical form, and no trace of the presumable spur-end cliffs of their early reefless stage: they have gained forms that are best explained on the supposition that, whether they past through an introductory reefless stage or not, they have long suffered dissection and intermittent subsidence, while protected from wave attack by an upgrowing, encircling barrier reef.

A theory of coral reefs is certainly commended when a reasonable extension of its postulated conditions and processes enables it to account for the cliffs of exceptional reef-encircled islands, for which no adequate explanation has been previously given.

ON SOME RELATIONS BETWEEN THE PROPER MOTIONS, RADIAL VELOCITIES, AND MAGNITUDES OF STARS OF CLASSES B AND A.

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Attention was called in a former paper[1] to a decrease of proper motion with decreasing brightness in the class B stars, whereas the radial velocities showed an increase. Such a condition was found also in the class A stars, but more pronounced. Owing to lack of proper-motion data for the other classes the matter was not pursued. Since those results were obtained I have seen Kapteyn and Adams' investigation[2] in which they find that the radial velocities increase with increasing proper motion for the spectral classes F, G, K and M. The inference from the magnitude-results for classes B and A had been that the contrary was probably true for these two classes. It seemed that in general large radial velocities would be associated with large proper motions, and *vice versa*.

After seeing Kapteyn and Adams' results I classified the B–B5 and A stars according to proper motion and found that in fact their velocities do appear to decrease slightly with increase of proper motion. As stated, the contrary was found by Kapteyn and Adams for the classes F, G, K and M. The B8–B9 stars were classified separately. These 45 stars appear to follow the later types with respect to proper-motion velocity characteristics if we use the V_1 values, or the earlier B and A types if we use V. It is perhaps significant that the B8–B9 stars have been found not to have a large K term.

The relations of proper motion and radial velocity for classes B–B5, B8–B9 and A are given in Tables I and II. Table III contains the proper motions arranged according to visual magnitude.

Table I reveals an apparent relation of some kind between the magnitudes and proper motions of these three classes. The faintest stars are found *near* (but not in) the stars of very smallest proper motions in all. In both directions from these the magnitudes decrease numerically (stars become brighter). The differences, however, are small and so far as the first group is concerned, particularly in class A, may be accidental, although examination indicates considerable consistency. The V_1 of class B–B5 and V of classes B8–B9 and A show a similar effect for the radial velocities—the largest being *near* (but not in) the very smallest proper motions. Very noticeable is the sudden increase

TABLE I*

Arranged according to proper motion

CLASS	μ	MEAN MAGNITUDE	MEAN V_1 km.	MEAN V km.	NUMBER OF STARS
B–B5.....	0".000 to 0".009	4.0	6.2	8.9	38
	0.010 0.019	4.5	7.9	7.6	44
	0.020 0.029	4.2	6.7	8.9	28
	0.030 0.049	3.9	5.5	8.0	49
	0.050 0.119	3.7	4.4	6.0	23
B8–B9.....	0.000 to 0.019	4.2	4.4	3.1	9
	0.020 0.049	4.8	6.7	7.2	16
	0.050 0.099	4.6	7.8	6.8	17
	0.100 0.180	3.0	11.1	6.6	3
A.....	0.000 to 0.020	4.5		10.1	31
	0.021 0.050	4.6		12.4	47
	0.051 0.099	4.5		11.1	60
	0.100 0.199	4.1		10.4	53
	0.200 0.656	3.0		10.1	18

* These tables are based upon Campbell's catalogs of B and A class stars in *Lick. Obs. Bulletin*, Nos. 196 and 211.

TABLE II

Arranged according to radial velocity

CLASS	V_2 km.	MEAN MAGNITUDE	MEAN μ	NUMBER OF STARS
B–B5.....	0 to 2.9	3.8	0.031	58
	3.0 5.9	4.0	0.024	47
	6.0 9.9	4.2	0.025	42
	10.0 19.9	4.4	0.029	24
	10.0 19.9	4.4	0.025*	23
	20.0 25.6	4.6	0.024	9
A.....	0 to 5.0	4.3	0.083	68
	5.1 9.9	4.2	0.104	49
	10.0 14.9	4.3	0.091	31
	15.0 19.9	4.4	0.112	27
	20.0 29.9	4.1	0.074	27
	20.0 29.9	4.5**	0.066	22
	30.0 35.5	4.7	0.068	8

* Omitting one large value.

** Omitting five bright stars.

TABLE III

	CLASS B–B5		CLASS A	
	Number of stars	Mean μ	Number of stars	Mean μ
1.9 and brighter	11	0".051	8	0".382
2.0 to 2.9	18	0.033	19	0.177
3.0 3.9	38	0.028	26	0.093
4.0 4.9	88	0.024	121	0.075
5.0 and fainter	24	0.025	38	0.066

of velocity in all three classes *near the smallest proper motions*. An examination of the results of Kapteyn and Adams indicates somewhat similar rather sudden increases in velocity in the classes F, G, and K, but in proper motions somewhat larger than in the B and A stars. Such an effect is not noticeable in the M stars. The application of a magnitude velocity correction would tend to reduce the peculiarity slightly.

As these velocities (V_1) for the B stars had been derived by including the constant error K, it was decided to see what the effect might be of neglecting it and using $V_1 = -19.5$ km., the same solar motions as were used in obtaining results for the A stars. The fifth column (V) contains these results. The general decrease of velocity with increasing proper motion is still indicated rather uncertainly but the sudden increase of velocity alluded to has been reversed in the B–B5 stars. This reversal seems to result chiefly from the application of K to the small negative velocities in the two groups.

It seems rather curious that this peculiar maximum in the velocities shifts about with the term K — where a considerable value has been found for it, the maximum and peculiar sudden increase of velocity are shown only if it is used. On the contrary, where no K term resulted, the maximum and sudden increase show with its omission. If confirmed, this indicates a relation of some sort.

The parallactic motion of the Sun is very noticeable in the proper motions of both the B and A classes and it seems not improbable that some systematic effect may have crept into the velocities when classified according to proper motions which have not been freed from the solar motion—that the peculiarities observed may be due to a preference of the faint stars and those of larger velocity for certain regions of sky.

The dependence of velocity upon magnitude is clearly shown in both classes B–B5 and A in the above classifications particularly in the residual radial velocities of table II. The effect of the few apparently abnormal velocities of bright stars in class A is here very marked also.

An examination of table II shows that in B–B5 stars with the possible exception of the smallest radial velocities (very brightest stars) there is little or no connection of the proper motion with the radial velocities. There is, however, a peculiarity in the case of the class A stars—the largest proper motions being associated with velocities of medium value. The largest velocities of all show the smallest proper motions—even after making liberal allowance for a magnitude-velocity effect. Until systematic effects such as magnitude-velocity equation, constant error, etc., are better understood, it seems useless to attempt a discussion of such small peculiarities. It seems probable, however, that

the velocity distributions in these two classes may not depart widely from Maxwell's law, but that they are different from those in the spectral classes F, G, K and M.

Conclusions.—1. The radial velocities of the stars of classes B and A appear to decrease in general with increasing proper motion. There are, however, some inexplained anomalies.

2. The velocity distributions of classes B–B5 and A differ from the distributions found for the F, G, K and M classes by Kapteyn and Adams, appearing to be more clearly in accord with Maxwell's law.

3. The effects of the magnitude-velocity equation are clearly shown in the classification of the B and A stars made for this investigation.

4. There are some indications of a small abrupt change in the velocities of both classes in the smaller proper motions. There are also some indications in the classes F, G and K of such a change but in proper motions somewhat larger than in the early classes.

[1] *Astrophys. J.*, **41**, 318.
[2] These PROCEEDINGS, **1**, 14.

ASYMMETRY IN THE PROPER MOTIONS AND RADIAL VELOCITIES OF STARS OF CLASS B AND THEIR POSSIBLE RELATION TO A MOTION OF ROTATION

By C. D. Perrine

OBSERVATORIO NACIONAL ARGENTINO, CÓRDOBA

Received by the Academy, April 8, 1916

In the course of other investigations asymmetry in the proper motions of the B stars was suspected. A preliminary investigation has shown a systematic difference between the proper motions in the region about 0^h and that opposite. The results of a comparison of the proper motions in these two regions and also with the regions within 40° of the apex and the antapex of solar motion are given in the table. The average radial velocities of the stars (V_2) are also included.

An examination of the individual values shows a marked consistency in all of the regions. These results depend upon Campbell's catalog of class B stars in *Lick Observatory Bulletin*, No. 196. As is well known these stars have a marked preference for the Milky Way.

The minima corresponding to the apices of the solar motion fall at $6\frac{1}{2}^h$ and 19^h. The two maxima corresponding to the parallactic motion of the Sun fall at 2^h and $13\frac{1}{2}^h$. These two points are almost exactly at right angles to the direction of the Sun's motion.

Region	*Apex*	*Antapex*	22^h–2^h	10^h–14^h	10^h–14^h[1]	10^h–14^h[2]
Magnitude	4.4	4.0	4.3	3.7	4.3	3.7
V_2 in km	7.8	6.2	6.8	5.9	7.1	6.0
Proper Motion	0".014	0".013	0".020	0".043	0".038	0".040
No. of stars	21	44	18	27	20	26

[1] Omitting all stars brighter than $3^M.0$ to make average magnitudes comparable with region at 22^h–2^h.

[2] Omitting the only apparently abnormal value of $\mu = 0".119$.

A cursory examination of the right ascensions of Boss' *Preliminary General Catalogue* reveals a consistently smaller number of very small proper motions in the region 10^h to 14^h than in the region 22^h to 2^h. A preliminary examination of the A stars shows a similar (but smaller) effect if we omit the stars having proper motions of $0".2$ and over.

At the time of finding asymmetry in these proper motions the determination by Charlier of the motion of the node of the invariable plane of the solar system on the plane of the Milky Way was unknown to me. The value for the motion derived by Charlier and the preliminary value obtained by me from the B stars are as follows:

Charlier (Boss *P. G. C.*) + $0".0035$ per year,
Perrine (class B) + 0.012 per year.

If the observed phenomenon is in reality a relative motion of the invariable plane with respect to the *Milky Way as a whole* the angular motion as derived from different classes of stars, but especially from stars at widely different distances, should be exactly the same. The above discordances in the values throw doubt on the constancy of that angle.

Preliminary solutions for the apex and velocity of solar motion from northern and southern stars separately of class B and of 110 stars brighter than 3.0 magnitude of all spectral classes give systematically different results. The solar velocities derived in this way have a particular bearing on the above question and are given below.

	Northern Stars km.	*Southern Stars* km.
Class B, 3.0 and fainter	−17.0	−24.8
2.9 and brighter, all spectral classes	−14.4	−24.5

If there is no motion of rotation the values of the solar velocity derived from these different regions should be the same except for the effects of accidental variations whether the invariable plane is in motion or not. If however, the stellar system is rotating in a retrograde direction about the poles of the Milky Way, exactly such differences as the above should result.

It seems highly probable that the invariable plane may also be in motion.

Conclusions.—1. The stars of class B show systematic differences in the proper motions in the two regions of the Milky Way at right angles to the direction of solar motion.

2. The solar velocities derived from the B stars in the northern and southern hemispheres separately differ, that from the northern stars being the smaller.

3. These conditions appear to be best explained by a general motion of rotation of the system of stars in a retrograde direction about an axis perpendicular to the Milky Way.

The details of these preliminary investigations will be published elsewhere.

THEORY OF AN AEROPLANE ENCOUNTERING GUSTS

By Edwin Bidwell Wilson

DEPARTMENT OF MATHEMATICS, MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Received by the Academy, April 10, 1916

A number of references to the theory of the stability of aeroplanes may be given,[1] but on their behavior in gusts little has been printed,[2] though much is probably known and held secret by foreign governments. Last summer I was asked to develop a theory of the effect of gusts upon (symmetric) aeroplanes, in particular upon a definite machine of which the aerodynamical coefficients were furnished me by Mr. Hunsaker. Two questions were to be answered: First, what is the effect of a gust upon the longitudinal motion in the uncontrolled machine; second, what is the effect when the machine is so constrained by an automatic device as to maintain its axis horizontal? My report, submitted to our National Advisory Committee for Aeronautics, will appear as a part of the *Report of the National Advisory Committee for Aeronautics, 1915* (Senate Document No. 268, 64th Congress, 1st Session). By the courtesy of the Committee I am permitted to give here a brief account of this work.

In treating the motion of an aeroplane the machine is referred to a set of moving axes x, y, z fixed in the body, and directed backward, to the left, and upward in normal horizontal flight with velocity $-U$. Departures from this standard condition of flight are treated by the method of small oscillations. If $U + u$, v, w, p, q, r be the linear and angular velocities in the disturbed motion and if u_1, v_1, w_1, p_1, q_1, r_1 be the linear and rotational velocities of the gusts (taken with the proper sign), the aerodynamic forces acting upon the machine will vary from those acting in the standard conditions by amounts which may be determined by the eighteen aerodynamic constants X_u, X_w, X_q, Z_u, Z_w, Z_q, M_u, M_w, M_q,

Y_v, Y_p, Y_r, L_v, L_p, L_r, N_v, N_p, N_r, of which the last nine refer to the 'lateral' motion which was not discussed.

The equations for the 'longitudinal' motion of the free machine are

$$\begin{aligned} du/dt - g\theta - X_u u - X_w w - X_q q &= X_u u_1 + X_w w_1 + X_q q_1, \\ dw/dt - Uq - Z_u u - Z_w w - Z_q q &= Z_u u_1 + Z_w w_1 + Z_q q_1, \\ B/M.\, dq/dt - M_u u - M_w w - M_q q &= M_u u_1 + M_w w_1 + M_q q_1, \end{aligned} \tag{1}$$

where θ is the inclination of the x-axis, and $B/M = k^2$ is the square of the radius of gyration about the y-axis. For small oscillations these equations are linear with constant coefficients. The solution of (1) may be carried out by the ordinary method. In particular the solution may be cast in such form that a number of different gusts may be discussed very rapidly after a certain amount of preliminary calculation has been made.

The data for the given machine are as follows:

$$\begin{aligned} X_u &= -0.128, & X_w &= 0.162, & X_q &= Z_q = M_u = 0, \\ Z_u &= -0.557, & Z_w &= -3.95, & M_w &= 1.74, \\ M_q &= -150, & k^2 &= 34, & U &= -115.5. \end{aligned}$$

The numerical equation for u is

$$\begin{aligned} (D^4 + 8.49\, D^3 + 24.5\, D^2 + 3.385\, D + 0.917)u &= \\ -(0.128\, D^3 + 1.16\, D^2 + 3.385\, D + 0.917)u_1 & \\ + (0.162\, D^2 + 0.715\, D + 1.647)\, Dw_1 - (59.37\, D + 560.6)\, q_1, & \end{aligned}$$

where $D = d/dt$, with similar results for w and θ. The roots of $D^4 + 8.49\, D^3 + 24.5\, D^2 + 3.385\, D + 0.917 = 0$ are $-4.18 \pm 2.43i$ and $-.0654 \pm .187i$. The solutions are therefore of the form $u = e^{-4.18t}(A \cos 2.43t + B \sin 2.43t) + e^{-.0654t}(C \cos .187\, t + D \sin .187t) + I_u$, where A, B, C, D are constants of integration and I_u is a particular integral obtained from some particular gust u_1, w_1, q_1. There are similar equations for w and θ. Of the twelve constants of integration only four are independent, and the relations between them are independent of the particular integrals I (provided the gust is not tuned to the free oscillation). It is therefore possible to set down, once for all, formulas for the coefficients, A, B, C, D in terms of the initial values of the particular integrals I_u, I_w, I_θ, I'_θ. Where the machine is in normal flight with $u = w = \theta = q = 0$.

The type of gust chosen was $J(1 - e^{-rt})$, where J is an intensity-factor. This gust rises from 0 to J in an infinite time, but the greater part of the rise occurs in the time $1/r$ or $2/r$. If we set u_1, w_1, or q_1 equal to $J(1 - e^{-rt})$, we obtain a head-gust, up-gust, or rotary gust. The

values 0.2, 1.0, 5.0 were successively assigned to r to correspond to mild, moderate, and sharp gusts.

The Uncontrolled Machine.—The result for a head-gust is that, during 15 to 18 seconds after striking the gust, the machine soars aloft to a height from $4J$ to $4\frac{1}{2}J$ above its initial level and subsequently executes a damped oscillation about the level $3\frac{1}{2}J$ above the original. The effect is not seriously affected by the sharpness of the gust (measured by r) but is proportional to its intensity (measured by J). The shock of the gust is small. For a rear-gust the effects are reversed in sign, the machine falls. A severe rear-gust coming upon a machine flying low might therefore produce serious consequences.

The major effect of an up-gust is to carry the aeroplane bodily upward with it. The oscillations are small, but in the case of a sharp gust, the initial acceleration may be considerable and 'bump' the pilot. The effects for down-gusts are reversed in sign.

Little appears to be known in regard to the extent or intensity or permanence of rotary aerial motions, and in the absence of such meteorological information, no satisfactory practical conclusions may be stated as to the effect of these gusts upon flight. The analysis, however, indicates that rotary gusts of relatively small intensity may have relatively large effects on the motion.

The Constrained Machine.—If the machine is so constrained that $\theta = 0$, the equations of motion reduce to the first two of (1). Substitution of the numerical data shows that the motion, when disturbed, is no longer oscillatory, but approaches the original condition asymptotically. The riding of the machine should therefore be steadier. The use of data obtained for lower speeds than $U = -115.5$ feet per second shows, however, that the constrained aeroplane loses its dynamical stability at a higher speed than the free machine, and to that extent is more dangerous in landing.

In a head-gust the constrained machine soars steadily (without oscillation) up to some J feet above its initial level. Thus the upward swoop of $4J$ to $4\frac{1}{2}J$ has been eliminated and the final increase in level is less than one-third as much as before. In a down-gust the effects are reversed in sign.

The up-gust is again important only for its general convective effect upon the machine. In case the gust is sharp, there may, however, be a considerable momentary acceleration or 'bump' at first.

It is interesting to note that if an aeroplane, equipped with the automatic device, is riding on a breeze which has a leisurely periodic gustiness, a habile pilot may 'suck' considerable energy out of the gusts. For

he may ride up in the head-gust, with the constraining device thrown off, to a height of some $4J$ and then, with the device working, need drop only about J in the rear-gust. In this manner he can gain the amount $3J$ in altitude during each period. To avoid interference the successive maxima of the gusts should be at least 30 seconds apart for this machine.

[1] G. H. Bryan, *Stability in Aviation*, Macmillan, 1910; L. Bairstow, *Technical Report of the Advisory Committee for Aeronautics*, London, 1912–13; J. C. Hunsaker, these PROCEEDINGS, 2, 278 (1916).

[2] A general lecture by Glazebrook, *Aeronautical Journal*, 272-301, July, 1914, should be cited.

TERMS OF RELATIONSHIP AND SOCIAL ORGANIZATION

By Truman Michelson

BUREAU OF AMERICAN ETHNOLOGY, WASHINGTON

Received by the Academy, April 17, 1916

Some years ago Kroeber undertook to show that terms of relationship are linguistic and psychological phenomena.[1] Recently Rivers has attempted to overthrow this view completely, and holds that they are sociological phenomena, and consequently that it is entirely possible to infer marriage customs and social organization from these terms.[2] Lowie lately has to a certain extent followed Rivers; he has not followed the latter's survival-theories,[3] and it is doubtful if many American ethnologists will do so.[4]

In this paper I wish to develop Kroeber's thesis from a different angle, and also to make a point on my own account. Long ago Morgan saw that for the most part the terms of relationships are identical in all Algonquian languages with phonetic changes, and consequently for the greater part must go back to the original parent language.[5] Now insofar as this is the case, to this extent terms of relationship are linguistic phenomena. For example, the Fox are organized in exogamic gentes with descent in the male line, and the Plains Cree have no gentile organization at all, yet have at least seventeen terms of relationship in common with the Fox. Again the Delaware who are organized in exogamic clans with descent in the female line have some terms of relationship in common with both the Fox and Cree. Similarly the Shawnee who have an entirely different organization from any of the above mentioned Algonquian tribes,[6] nevertheless have many terms in common with Plains Cree and Fox, and some with Delaware. Accordingly it is obvious that social organization is not the sole factor in terms of relationship. It may be objected that though Plains Cree and Fox possess

many terms in common, yet they are not all used with the same meaning, and that the difference in social organization is responsible for this. For example Plains Cree and Fox have a term in common for 'my daughter,' but in Fox a male speaker can apply it to his brother's daughter also, whereas in Plains Cree he cannot. To this we reply that though both the Ojibwa and Fox are organized in exogamic gentes with descent in the male line and possess a number of terms of relationship in common, nevertheless all the terms are not used with the same meaning. An example is the term for 'my father' which is common to both Fox and Ojibwa as well as to numerous other Algonquian tribes, but which can be used in Fox also with the sense of 'my paternal uncle' though it cannot be so used in Ojibwa where an entirely distinct word is used for this latter meaning. So it is clear that difference in social organization by itself will not satisfactorily account for the different usage of the same term in Plains Cree and Fox. When it is further noted that Ojibwa, Ottawa, Algonkin, Potawatomi, and Cree share in common words for 'my brother's son' and 'my brother's daughter,' with a male speaker in each case, as opposed to Sauk, Fox, Kickapoo, Shawnee, Menominee, Peoria and closely allied dialects—all of whom employ the common Algonquian words for 'my son' and 'my daughter' respectively for the above— it becomes clear that we have to deal with linguistic phenomena. For it will be remembered that Cree-Montagnais, Menominee, Sauk, Fox, Kickapoo, and Shawnee, form one sub-division of Central Algonquian dialects and Ojibwa, Ottawa, Algonkin, Potawatomi, Peoria and closely related dialects another such sub-division.[7] Adequately to account for the divergent distribution of the terms in question we must assume that Cree has bodily borrowed them from Ojibwa, and that Peoria and closely related dialects under the influence of Sauk, Fox, etc., have extended the meanings of the terms for 'my son' and 'my daughter' to include also 'my brother's son' and 'my brother's daughter' (with male speaker in each case) respectively. In other words, in the latter case a category has been taken over. The geographical distribution of the tribes mentioned distinctly favors these hypotheses. It is another linguistic question as to which of the two sets of terms represents the hypothetical Algonquian parent-language most closely. The fact that Delaware and Munsee, who are organized in exogamic clans with descent in the female line, and who form a distinct sub-division of the Central Algonquian dialects, and Micmac, a somewhat divergent member of Eastern Algonquian dialects, agree with Sauk, Fox, etc., tends to show that the latter represent the primitive

Algonquian state as opposed to Ojibwa, etc.[8] This is exactly what might have been anticipated, for the sub-division represented by Cree-Montagnais, Menominee, Sauk, Fox, Kickapoo, and Shawnee in many respects linguistically is more archaic than the subdivision represented by Ojibwa, Ottawa, Algonkin, Potawatomi, Peoria and closely related tribes.[7] This applies especially to the first four dialects of the latter sub-division.

I spoke above of the borrowing of categories to explain certain terms of relationship. But this was within a single linguistic stock. Now as some members of both the Iroquoian and Siouan stocks have systems of consanguinity that in certain respects are identical with those of some members of the Algonquian stock, and at the same time geographically are contiguous to them, I do not think it likely that this is merely the result of accident. In short we have borrowing of categories across linguistic stocks. I mention this to show how inadequately social organization by itself can account for the terms of relationship of Algonquian languages.

Again the Piegan, a member of a linguistically divergent major group of Algonquian tribes, who are apparently organized in gentes in the making, have practically no terms of relationship that are the phonetic equivalents of those of the central group. Nor do the categories correspond with theirs. It is difficult to see how this is due to the social organization of the former. It will be borne in mind that the Plains Cree are if anything looser in organization, and yet have the old terms of relationship. It will be recalled that Cree in many respects is an archaic Algonquian language.[7] On the face of it, it looks as if we again had a linguistic problem.

Summing up, we must say that from the point of view of Algonquian tribes terms of relationship are linguistic and disseminative phenomena. But I do not deny that in other cases they may be primarily psychological and sociological. We have no right to generalize from a single stock and apply our conclusions everywhere. When some dozens of stocks shall have been analyzed somewhat on the above plan, it may be possible safely to generalize. Till then we have every reason to believe that all the factors have played a rôle in kinship-terms.

[1] *J. R. Anthrop. Inst. G. B. I., London*, 39, 77–84.

[2] *Kinship and Social Organization*, London, 1914; *The History of Melanesian Society*, Cambridge, England, 1914.

[3] These PROCEEDINGS 1, 346–349; *Amer. Anthrop., New York*, N. S., 17, 223–239. However Lowie makes an exception in the case of the Cree terms of relationship which, he thinks,

points to the fact that the Cree once possessed exogamous gentes with descent in the male line. How unnecessary this hypothesis is, is shown in this paper.

[4] See *Amer. Anthrop., New York*, N. S., 17, 329–340; *Ibid.*, 588–591; per contra, *Ibid.*, 175–177.

[5] *Smithsonian Contributions to Knowledge*, No. 218, p. 217.

[6] According to my unpublished information.

[7] *Smithsonian Inst., Rep. Bur. Amer. Ethn.*, 28, 221–290b.

[8] We are confirmed in this belief by the fact taat the Atsina, a member of a linguistically divergent major group of Algonquain tribes, organized in non-totemic exogamous gentes with descent in the male line employ the same words for 'my son' and 'my daughter' as they do for for 'my brother's son' and 'my brother's daughter' respectively, with male speaker in the latter cases. The word for 'my daughter' is the common Algonquian one; that for 'my son' apparently is not old.

REPORT OF THE ANNUAL MEETING

Prepared by the Home Secretary

The sessions of the Annual Meeting of the National Academy of Sciences were held in the United States National Museum, Washington, D. C., on April 17, 18, and 19, 1916.

Seventy-two members were present as follows: Messrs. C. G. Abbot, Allen, Ames, Becker, Boltwood, Britton, Bumstead, D. H. Campbell, Cannon, Cattell, W. B. Clark, F. W. Clarke, F. M. Clarke, Comstock, Conklin, Coulter, Cross, Dall, Davenport, Davis, Day, Donaldson, Fewkes, Flexner, Hague, Hale, E. H. Hall, Harper, Hayford, Hillebrand, Holmes, Howell, Iddings, Jennings, Leuschner, Lillie, Loeb, Lusk, Mall, Meltzer, Mendel, Merriam, Merritt, Milliken, More, Morley, H. N. Morse, Moulton, E. L. Nichols, A. A. Noyes, Parker, Pickering, Pupin, Ransome, Reid, Remsen, Rosa, Schuchert, Scott, E. F. Smith, Story, Van Hise, Van Vleck, Walcott, Webster, Welch Wheeler, David White, H. S. White, Wilson, R. W. Wood, Woodward.

BUSINESS SESSIONS

The President announced the following deaths since the last Annual Meeting of the Academy:

JOHN ULRIC NEF, died August 13, 1915, elected in 1904.

FREDERICK WARD PUTNAM, died August 18, 1915, elected in 1885.

ARTHUR W. WRIGHT, died December 19, 1915, elected in 1881.

EUGENE W. HILGARD, died January 8, 1916, elected in 1872.

PAUL EHRLICH, foreign associate, died August 20, 1915.

REPORTS FROM OFFICERS OF THE ACADEMY

The reports of the President and Treasurer for the year 1915 were presented to the Academy in printed form as transmitted to the Senate of the United States by the President of the Academy.

The report of the Home Secretary was presented as follows:

The President of the National Academy of Sciences.

Sir: I have the honor to present the following report on the publications and memberships of the National Academy of Sciences for the year ending April 19, 1916.

The Memoirs of the National Academy of Sciences, volume 12, part 2, entitled, "Variations and Ecological Distribution of the Snails of the Genus Io," by Charles C. Adams, has been published and distributed, as has also the memoir forming volume 12, being, "A Catalogue of the Meteorites of North America," by Oliver C. Farrington. Volume 14, memoir 1, entitled, "Report on Researches on the Chemical and Mineralogical Composition of Meteorites, with Especial Reference to Their Minor Constituents," by George Perkins Merrill, is going through the press and the final proof has been passed. It awaits casting and printing before it is published.

The biographical memoirs of John W. Powell, Miers Fisher Longstreth, Charles Anthony Schott, Peter Lesley, Henry Morton, and Alfred Marshall Mayer have been published, and that of George William Hill, by Ernest W. Brown, has also been published but not distributed.

Four members have died since the last Annual Meeting: John Ulric Nef, on August 13, 1915, elected in 1904; Frederick W. Putnam, on August 18, 1915, elected in 1885; Arthur W. Wright, on December 19, 1915, elected in 1881; and Eugene W. Hilgard, on January 8, 1916, elected in 1872.

One foreign associate, Paul Ehrlich, died on August 20, 1915.

There are 139 active members on the membership list, 1 honorary member, and 38 foreign associates.

Respectfully submitted, ARTHUR L. DAY, Home Secretary.

The report of the Foreign Secretary was presented as follows:

I have the honor to report on the work of the foreign secretary for the year ending April 19, 1916.

An attempt has been made, through correspondence with various academies and societies belonging to the International Association of Academies, to secure a partial continuance of some portions of the Association's work through the period of the war. Although international meetings are obviously not feasible, it was hoped that a temporary transfer of the functions of the leading academy from Berlin to Amsterdam, as suggested by the former body, might serve a useful purpose. Unfortunately, however, certain difficulties of an insuperable nature prevented the proposed transfer, and no further steps can be taken at present.

It was suggested to the Amsterdam Academy by the foreign secretary, also acting in the capacity of secretary of a joint committee of the National Academy and the American Association for the Advancement of Science, that the Accademia dei Lincei be requested to use its good offices to secure the continuation of the work of the Zoological Station at Naples. A favorable reply was received from the President of the Lincei; but the participation of Italy in the war has prevented Dr. Dohrn from retaining the direction of the Station, which is now under an Italian administration.

At the request of the president of the Amsterdam Academy, who is also permanent secretary of the International Geodetic Association, the Secretary of State was asked by the Academy to use his influence to secure the continued participation of the United States in the work of the Association, and the maintenance of the international latitude station at the Ukiah, California. Through the action of the Secretary of State, and the interest of members of Congress, the necessary appropriations have been provided.

GEORGE E. HALE, Foreign Secretary.

REPORTS FROM COMMITTEES ON TRUST FUNDS

A report was received from the directors of the Bache Fund, stating that Ira Remsen resigned as Director of the Fund at the annual meeting, 1915, and

that the two remaining members of the committee chose Arthur G. Webster as the third member and that Edwin B. Frost was elected chairman. This report contained also an announcement of the research grants made from the Bache Fund during the year ending April 17, 1916; and it stated that on April 7, 1916, the Bache Fund had on hand a cash income balance of $980.62, and an invested income of $2575.

A report was received from the Trustees of the Watson Fund, signed by Edward C. Pickering (chairman) and W. L. Elkin. In the report the trustees recommended that the sum of $500 from the income of the Watson Fund be appropriated to Professor John A. Miller, Director of the Sproul Observatory, for measuring plates already taken for the determination of stellar parallaxes. This is a continuation of a grant awarded last year. It was recommended that the sum of $300 from the income of the Watson Fund be appropriated to Professor Herbert C. Wilson, Director of the Goodsell Observatory, for measurements of the positions of asteroids on photographs already taken. These recommendations were adopted by the Academy.

A report was received from the Committee on the Henry Draper Fund, signed by George E. Hale (chairman) as follows:

> Four members of the committee, without consulting the fifth member (Professor Michelson), recommend that the Henry Draper Gold Medal be awarded to ALBERT A. MICHELSON, of the University of Chicago, for his numerous and important contributions to spectroscopy and astronomical physics.
>
> It is impossible in the brief space of this report even to enumerate Professor Michelson's major services to science. These include the precise determination of the velocity of light; the well-known experiment (with Professor Morley) on ether drift; the measurement of the absolute wave-length of light involved in his determination of the length of the standard meter; the measurement of tides in the body of the earth with new apparatus of extraordinary precision; and the invention of the interferometer, the echelon, and other instruments of prime importance to the student of light. He has also constructed a ruling machine yielding diffraction gratings of the longest size and the highest resolving power yet attained, and carried on a multiplicity of researches of wide range and fundamental significance.
>
> The committee also recommends that a grant of $250 be made to Professor Philip Fox, Director of the Dearborn Observatory, of Northwestern University, Evanston, Illinois, to apply toward the cost of a machine for measuring astronomical photographs.

The recommendations contained in the report were adopted by the Academy.

A report was received from the Committee on the J. Lawrence Smith Fund, signed by Edward W. Morley (chairman). A brief account was given of the researches aided by previous grants. The Committee reported a cash balance of income of $834.77, of which $250 is already appropriated, though not yet paid over. There is also an invested income balance of $1532.50. The Committee unanimously recommended that a further grant of $300 be made to S. A. Mitchell, professor in the University of Virginia, University, Va., to aid in securing observations of paths and of radiants of meteors and in computing orbits where observations are sufficient. This recommendation was adopted by the Academy.

A report was received from the Committee on the Comstock Fund, signed by Edw. L. Nichols (chairman). The Committee reported that the total income from the fund now available is $1661.32. The next award will be made at the end of the five-year period specified in the bequest, namely, at the annual meeting in April, 1918.

A report was received from the Directors of the Walcott Gibbs Fund, signed by the three directors. The report stated that T. W. Richards was elected to fill the vacancy caused by the withdrawal of Ira Remsen from the board. Only one appropriation had been made from the income of the Fund—a grant (No. 6) to Professor Gregory P. Baxter of Cambridge of $300 to provide apparatus, especially of platinum and quartz, and materials for his researches on atomic weights and changes of volume during solution. The unexpended income of the fund was stated to be $90.77. A brief account was given of the progress made on the researches aided by previous grants.

A report was received from the Committee on the Murray Fund, signed by Arnold Hague (chairman). The Committee announced that the unusual expenses due to the designing and striking off of the Agassiz medal, as called for by the terms of the gift, has required all the early income; but that from now on the interest from the fund will be applied for the striking off of the Agassiz medal and contributions to oceanography.

A summarized statement of the grants made from the various trust-funds of the Academy during the year ending April 17, 1916, will be found at the end of this number of the PROCEEDINGS.

REPORT OF THE BOARD OF EDITORS OF THE PROCEEDINGS

The Board of Editors of the PROCEEDINGS reported to the Academy as follows:

The twelve numbers of the PROCEEDINGS issued since its last annual report contain 157 original papers, the report of the Annual and Autumn Meetings, notices of scientific memoirs, and the report of the Panama Canal Committee. These numbers consist of 641 pages, an average of 53 pages per number, and of about 4 pages per article. The papers are distributed among different sciences as follows: mathematics, 17, astronomy 37, physics 11, chemistry 12, geology, paleontology, and mineralogy 16, botany 3, zoology 13, genetics 17, physiology, pathology, bacteriology, and biochemistry 16, anthropology 13, psychology 2.

The universities or research laboratories that have contributed three or more papers are as follows: Carnegie Institution 32, divided as follows: Mount Wilson Solar Observatory 22, Nutrition Laboratory 4, Marine Biology 3, Terrestrial Magnetism 1, Botanical Research 1, Geophysical Laboratory, 1; University of California 17; Harvard University 16; Johns Hopkins University 14; University of Chicago 13; Yale University 7; Rockefeller Institute 7; Princeton University 6; U. S. Geological Survey 5; U. S. National Museum 5; Station for Experimental Evolution 4; University of Illinois 3; American Museum of Natural History 3.

The edition of the PROCEEDINGS was reduced from 3000 to 2200 copies with the issue of May, 1915, and it is to be further reduced to 1700 copies with the issue of May, 1916, as the distribution of free copies in America is to be now entirely discontinued. At present about 900 copies are mailed to foreign exchanges and about 250 copies to members of the Academy and other American subscribers.

The total expense incurred in connection with the PROCEEDINGS during the year 1915, representing approximately the cost of publication of the first volume, is shown by the report of the treasurer to be $4300.67. Of this sum $2977.37 was expended for printing and distributing, corresponding to a cost of $4.65 per page.

The Board recommends that action be taken by the Academy informing members and others presenting accounts of new researches at the Academy meetings that they are expected to furnish to the Home Secretary at the time of the meeting a brief paper describing these researches in form suitable for publication in the PROCEEDINGS. The Board also suggests that, in order to secure fuller circulation of the PROCEEDINGS in our universities, colleges, and research laboratories members of the Academy be requested to see that their own institutions subscribe to the PROCEEDINGS in case they have not already done so.

GENERAL BUSINESS

A report of the Committee on Bill H. R. 528, discontinuing the use of the Fahrenheit thermometer scale in government publications was adopted as follows:

Your committee for the consideration of Bill H. R. 528, consisting of Messrs. C. G. Abbot, S. W. Stratton and C. M. Marvin, unanimously reports the following resolution, and moves its adoption.

The National Academy of Sciences shares the desire of scientific men in general for international and world-wide uniformity in units of measurement of all kinds, and with this object in view it favors the introduction of the Centigrade scale of temperature, and units of the metric system generally, as standards in the publications of the United States Government.

It must be recognized that considerable initial expense must be incurred by the U. S. Weather Bureau in changing its apparatus to conform to the proposed act. Furthermore, on account of the more open scale of the Centigrade system that Bureau will be subject to a continued increased cost of publication, owing to the necessity of printing the first decimal place in order to maintain the present accuracy. The use of negative temperatures and minus signs entails greater liability to errors, and more clerical labor would be required in checking the accuracy of the reports of coöperative observers of the Weather Bureau, and in computing monthly and other mean temperatures.

Notwithstanding the foregoing, the Academy is in favor of legislation to make the Centigrade scale of temperatures the standard in publications of the United States Government, and funds should be made available by Congress to accomplish the desired result.

The Academy favors Bill H. R. 528, "To discontinue the use of the Fahrenheit Thermometer Scale in Government Publications," but recommends that it be amended by the addition of the following:

"Sec. 4. When in the publication of tables containing several meteorological and climatic elements, the use of data in Centigrade temperatures leads to manifest incongruities, the Chief of the Weather Bureau is directed to publish related data in such units as are necessary to make the tables homogeneous and to secure international uniformity as far as practicable.

"Sec. 5. Nothing in this act shall prevent the use of the absolute Centigrade scale of temperature in publications of the Government."

Upon recommendation of the Council it was voted that in accordance with the request of the chairman of the Committee on Foreign Affairs of the House of Representatives a committee of the Academy be appointed to prepare a report upon the joint Resolution (H. J. Res. 99), "That the President be, and he is hereby, requested to ascertain the views of foreign governments regard-

ing the proposition to appoint an international commission to prepare a universal alphabet," and that the report be submitted to the President of the Academy, who in turn will transmit it to the chairman of the Committee on Foreign Affairs of the House of Representatives, reporting his action in the matter at the next annual meeting of the Academy.

Messrs. Dewey, Bell, Lindgren, Cattell, and Boas were appointed members of this committee.

The Council also recommended to the Academy the appointment of a committee to discuss possible plans of coöperation with a committee of engineers. A committee was appointed, consisting of George E. Hale, chairman; J. S. Ames, John F. Hayford, E. L. Nichols, M. I. Pupin, E. B. Rosa, Elihu Thomson, C. R. Van Hise, C. D. Walcott, and R. S. Woodward.

The President announced that an invitation had been received from the members of the Academy living in Boston that the Academy hold its Autumn Meeting in the year 1916 in that city. The following members were appointed to serve as a local committee of this meeting: William M. Davis, chairman, W. T. Councilman, Arthur A. Noyes, George H. Parker and E. C. Pickering.

ELECTION OF FOREIGN SECRETARY, COUNCILLORS AND NEW MEMBERS

Mr. GEORGE E. HALE was reelected foreign secretary of the Academy for a term of six years.

R. H. CHITTENDEN and M. I. PUPIN were elected members of the Council for a term of three years.

The following persons were elected to membership in the Academy:

GILBERT AMES BLISS, mathematician, University of Chicago, Chicago, Illinois.
FRANK SCHLESINGER, astronomer, University of Pittsburgh, Pittsburgh, Pa.
GREGORY PAUL BAXTER, chemist, Harvard University, Cambridge, Mass.
MARSTON TAYLOR BOGERT, chemist, Columbia University, New York City.
LELAND OSSIAN HOWARD, entomologist, U. S. Department of Agriculture, Washington, D. C.
ALFRED GOLDSBOROUGH MAYER, zoologist, Carnegie Institution, Tortugas, Florida.
RAYMOND PEARL, biologist, Maine Agricultural Experiment Station, Orono, Maine.
PHOEBUS AARON THEODORE LEVENE, physiological chemist, Rockefeller Institute for Medical Research, New York City.
OTTO FOLIN, physiological chemist, Harvard Medical School, Boston, Mass.

SCIENTIFIC SESSIONS

Two public lectures on the WILLIAM ELLERY HALE FOUNDATION were given on April 17 and 19 by HENRY FAIRFIELD OSBORN, President of the American Museum of Natural History, on The Origin and Evolution of Life on the Earth.

Four public scientific sessions were held on April 17 and 18 at which the following papers were presented:

SYMPOSIUM ON THE EXPLORATION OF THE PACIFIC. ARRANGED BY W. M. DAVIS

W. M. DAVIS: On exploration of the Pacific.
J. F. HAYFORD: The importance of gravity observations at sea in the Pacific.

L. J. BRIGGS, President of the Philosophical Society of Washington: A new method of determining gravity at sea.

C. SCHUCHERT: The problem of continental fracturing and diastrophism in Oceanica.

J. P. IDDINGS: Petrological problems in the Pacific.

G. W. LITTLEHALES: Hydrographic Engineer United States Hydrographic Office: The extent of knowledge of the oceanography of the Pacific.

C. F. MARVIN, Chief of the United States Weather Bureau: Marine meteorology and the general circulation of the atmosphere.

WM. H. DALL: On the distribution of Pacific invertebrates.

H. A. PILSBURY, Academy of Natural Sciences of Philadelphia: Land mollusca of the Pacific.

W. G. FARLOW: Marine algae of the Pacific.

D. H. CAMPBELL: Problems of the Pacific floras.

J. W. FEWKES: The Pacific as a field for anthropological investigation.

REGULAR PROGRAM

S. J. MELTZER: On permeability of endothelia.

I. S. KLEINER and S. J. MELTZER: The influence of morphin upon the elimination of intraveneously injected dextrose.

JACQUES LOEB: The sex of a parthenogenetic frog.

EDMUND B. WILSON: The distribution of the chondrisomes to the spermatozoa in scorpions.

ARTHUR KEITH (introduced by George F. Becker); A new form of metamorphism.

J. P. IDDINGS and E. W. MORLEY: Contributions to the Petrology of Japan, Philippine Islands and the Dutch Indies.

CHARLES R. STOCKARD, Professor of Anatomy, Cornell University Medical College: Hereditary transmission of defects resulting from alcoholism.

W. B. CANNON: Recent observations on the activity of some glands of internal secretion.

H. H. DONALDSON: Studies in the water content of the nervous system.

GEORGE E. HALE: Some recent results of solar research.

CHARLES E. ST. JOHN (introduced by G. E. Hale): An investigation of the suggested mutual repulsion of Fraunhofer lines.

ARTHUR S. KING (introduced by G. E. Hale): Anomalous dispersion phenomena in electric furnace spectra.

WALTER S. ADAMS (introduced by G. E. Hale): Illustrations of the new spectroscopic method of measuring stellar distances.

HARLOW SHAPLEY (introduced by G. E. Hale): Some results with the new 10-inch photographic telescope.

C. G. ABBOT and L. B. ALDRICH: The pyranometer, an instrument for the measurement of sky radiation.

G. C. COMSTOCK: Invisible companions of binary stars.

EDWIN H. HALL: Theory of electric conduction in metals.

F. R. MOULTON: The evolution of the stars.

A. O. LEUSCHNER, Watson Medallist: The minor planets discovered by James C. Watson.

WM. H. DALL: Biography of Professor Theodore Nicholas Gill (by title).

W. W. CAMPBELL: Biography of Professor Edward Singleton Holden (by title).

W. W. CAMPBELL: Biography of Professor Simon Newcomb (by title).

FIELDING H. GARRISON: Biography of John Shaw Billings (by title).

CHARLES R. VAN HISE: Report of the work of the committee upon the Panama Canal slides.

H. FIELDING REID: The mechanics of the Panama slides.

THEODORE LYMAN, Director Jefferson Physical Laboratory, Harvard University: The present state of knowledge of the extreme ultra violet.

ROBERT A. MILLIKAN: A redetermination of e and N.

CARL L. ALSBERG, Chief of the Bureau of Chemistry, United States Department of Agriculture: The relation of investigational work to the enforcement of the food and drugs act.

J. WALTER FEWKES: Recent explorations on the Mesa Verde National Park, Colorado.

ERWIN F. SMITH: Further evidence on the nature of crown gall and cancer and that cancer in plants offers strong presumptive evidence both of the parasitic origin and of the essential unity of the various forms of cancer in man and animals.

AWARD OF MEDALS

At the annual dinner of the Academy held at the Hotel Raleigh on April 18, 1916, Medals for Eminence in the Application of Science to the Public Welfare were awarded to Cleveland Abbe for distinguished public service in establishing and organizing the weather service of the United States, and to Gifford Pinchot for distinguished public service in organizing and directing the movement for the systematic conservation of the natural resources of the United States; and the James Craig Watson medal was awarded to Armin Otto Leuschner for the skill and ability which he has shown in supervising the preparation of tables of the Watson asteroids, involving original methods, and leading to results of much value to celestial mechanics.

RESEARCH GRANTS FROM THE TRUST FUNDS OF THE ACADEMY

During the twelve months preceding the Annual Meeting of the Academy the following grants for the promotion of research were made from the trust funds of the Academy.

GRANTS FROM THE BACHE FUND

No. 188, H. W. NORRIS, Grinnell College, $100. For assistance in the analysis of the cranial nerves of Coecilians (Herpele and Dermophis).

No. 189, E. J. WERBER, Woods Hole, $230. For assistance in experimental studies aiming at the control of defective and monstrous development: (1) the effect of toxic products of metabolism on the developing teleost egg; (2) the effect of experimentally produced diseases of parental metabolism on the offspring of mammals.

No. 190, H. S. JENNINGS, Johns Hopkins University, $200. For assistance in the study of evolution in a unicellular animal multiplying by fission: heredity, variation, racial differentiation in *Difflugia*.

No. 191, P. W. BRIDGMAN, Harvard University, $500. For mechanical assistance in an investigation of various effects of high hydrostatic pressure, in particular the effect of pressure on electrical resistance of metals. (Supplementary to grant No. 184.)

No. 192, J. P. IDDINGS, Washington, D. C., $1000. For apparatus and assistance in the microscopical and chemical investigation of igneous rocks for the purpose of extending knowledge regarding petrographical provinces and their bearings on the problem of isostasy.

No. 193, C. A. KOFOID, University of California, $500. For assistance in securing animals in the Indian jungle and in their preparation for study in research on the intestinal protozoa.

No. 194, R. A. DALY, Harvard University, $1000. For the purchase of a thermograph of new design for determining temperatures in the deep sea.

No. 195, R. W. HEGNER, University of Michigan, $160. For assistance in the study of the history of the germ cells, especially in hermaphrodite animals, in order to determine the visible changes that take place in their differentiation and the causes of these changes. (Supplementary to grant No. 185).

GRANT FROM THE J. LAWRENCE SMITH FUND

No. 6, S. A. MITCHELL, University of Virginia, University, Va., $500. For aid in securing observations of paths and of radiants of meteors and in computing orbits where observations are sufficient.

GRANT FROM THE WOLCOTT GIBBS FUND

No. 6, GREGORY P. BAXTER, Harvard University, $300. To provide apparatus, especially of platinum and quartz, and materials for his researches on atomic weights and changes of volume during solution.

The following additional grants were authorized by the Academy at the business sessions of April 17–19, 1916.

GRANT FROM THE HENRY DRAPER FUND

PHILIP FOX, Dearborn University, Northwestern University, $250. To be applied towards the cost of a machine for measuring astronomical photographs.

GRANTS FROM THE WATSON FUND

No. 12, HERBERT C. WILSON, Goodsell Observatory, $300. For measurements of the position of asteroids on photographs already taken.

No. 13, JOHN A. MILLER, Sproul Observatory, Swarthmore College, $500. For measuring plates already taken for the determination of stellar parallaxes. (Supplementary to grant No. 10.)

GRANTS FROM THE BENJAMIN APTHORP GOULD FUND

S. D. TOWNLEY, Lick Observatory, $200. For observing variable stars.

R. W. WOOD, Johns Hopkins University, $200. For apparatus for photographing celestial bodies with monochromatic light.

A. VAN MAANEN, Mount Wilson Solar Observatory, $100. For clerical assistance in the reduction of places of certain propermotion stars.

EDWIN B. FROST, Yerkes Observatory, $640. For a measuring machine for measuring parallax plates.